CELEBRATIO MATHEMATICA

Michael F. Atiyah

Nigel Hitchin

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The early years

Michael Atiyah was born in London on 22 April 1929. His father Edward, from a Lebanese family, was working in the political service in the Sudan at the time, though he was also an author and broadcaster. His mother Jean (née Levens), of Scottish descent, grew up in Oxford where she studied art and where she also met her husband. As a child, Michael lived in Khartoum, attending the Diocesan School there, but returned to England for long periods during the spring and summer to avoid the heat.

His secondary education began in 1941, travelling to Lebanon via France just before its collapse. Very soon afterwards he left the French school in Lebanon and enrolled in Victoria College, an English-style boarding school in Cairo where his father had been. In this cosmopolitan community, he fared well in mathematics, staying a couple of years ahead of his age group, but became perhaps more interested in chemistry as he progressed through the school. Ultimately, he was vanquished by the feats of memory which chemistry demanded, and reverted to the mathematics which his parents always felt was his real talent.

Having at the age of 16 decided on mathematics as a subject to take further, the Atiyahs' thoughts naturally turned towards Cambridge; to prepare for this, their son was sent to Manchester Grammar School in 1945. He had already taken his A-level examinations, and was quite confident and not prepared for the hothouse atmosphere of mathematics at the school, which showed that the subject had more depths than he had been aware of. However, under the tutelage of a dedicated mathematics master, who only used 19th century books and produced his own notes for everything more recent, he won a scholarship to Trinity College, accepted in Cambridge along with a sizable cohort of mathematicians from the same school.

At that point, however, the outside world intervened in the form of military service. This could have been postponed, but Michael deliberately chose to do it then and get it out of the way; so, after leaving school at Easter 1947, he joined the Royal Electrical and Mechanical Engineers. As a Cold Warrior he was confined to clerical duties at the regimental Headquarters, but it was an occupation that gave him spare time, which he spent reading mathematics books like Hardy and Wright, and doing the problems sent on from Cambridge by his school contemporaries who had gone there directly. Fortunately, a well-timed

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letter from his tutor enabled him to leave the army early in 1949 and finally reach his goal of Cambridge for the long vacation term.

Cambridge

After eighteen months of National Service, Cambridge was a liberation. His friends who went directly from school were already getting a little exhausted, whereas he felt full of enthusiasm, and immersed himself in reading in Trinity College library. When term actually began in October, he found himself being supervised in analysis by the idiosyncratic A. S. Besicovitch and in geometry by the more staid J. A. Todd. It was the geometry — classical geometry — that he enjoyed most and, indeed, was the source of his first mathematical paper, published in the *Proceedings of the Cambridge Philosophical Society* in his second year as an undergraduate. Michael's undergraduate years were spent in going to as many lectures as he could physically manage and in discussing and arguing with his mathematical friends, who included J. F. Adams, I. G. Macdonald and J. Polkinghorne, who became a well-known physicist, and also J. P. H. Mackay, the future Lord High Chancellor of Great Britain. Michael and John Polkinghorne prepared themselves intensively for the examinations, going through old papers in the library on Saturday mornings, and ended up taking the top honours in the end-of-year examination.

By this time, however, he was already thinking about going further in mathematics. Having written a paper, attended many Part III courses and persuaded W. V. D. Hodge to lecture on topics he wanted to hear, it was clear that mathematical research was his goal and, forsaking Todd, he became the research student of Hodge. Unusually, Hodge had four research students that year (1952). Besides Michael, these included R. Penrose, who had arrived from University College London and later moved to Todd as a supervisor, and M. Hoskin, now a historian of science. The fourth student left to become a schoolteacher.

Hodge put his new student to work on characteristic classes, reading the papers of S.-S. Chern, A. Weil and C. B. Allendoerfer. This sort of global differential geometry was novel at that time, and although courses in topology by P. Hilton and S. Wylie helped it was not, despite Hodge's book on harmonic integrals a decade earlier, a subject easy to find out about in Cambridge. It was through his weekly reading of the *Comptes Rendus* that Michael began to absorb the new results in sheaf theory that were being produced by the French school at that time, and influenced his research at this stage. A sabbatical visit to Cambridge by N. S. Hawley focused his attention on problems concerning holomorphic vector bundles, and in beginning to work in this area he discovered a fundamental misconception in one of Hawley's papers. Resolving this gave rise to the research paper on ruled surfaces and holomorphic projective bundles, which won Michael the Smith's prize in 1954. Shortly afterwards, he collaborated with Hodge using sheaf theory to study integrals of the second kind. At the end of that year he won a Fellowship to Trinity College.

The award of the prize gave him the boost to stay in mathematics and not leave the academic environment, as many of his contemporaries did, some to national security work at GCHQ, others like Ian Macdonald to the Civil Service. The International Congress at Amsterdam in 1954, with K. Kodaira and J-P. Serre receiving Fields Medals, fired his imagination even more as he saw the vista of possibilities in just the sort of mathematics in which he was working.

Princeton

The year 1955 was an uneventful, unproductive one. Having seen the mathematical stars at the ICM, Cambridge was far too quiet for Michael, and following Hodge's visit to Princeton the previous year, where he talked about their joint work, he jumped at D. C. Spencer's invitation to go there too. On receiving a Commonwealth Fellowship he had the opportunity to go to the Institute for Advanced Study rather than the University, which he readily took with Spencer's encouragement. Before doing so, however, he married Lily Brown, who had been studying mathematics a year ahead of him in Cambridge. She gave up her job at Bedford College London to join him on this, his first, visit to the United States.

If Cambridge was an eye-opener after National Service, Princeton was even more so after Cambridge. The Institute under R. Oppenheimer had a big concentration of visitors, with several overlapping generations. R. Bott and Serre were there, as was I. M. Singer. At the University, F. Hirzebruch was a professor. Bott lectured on Morse theory, Kodaira on sheaf theory, currents and algebraic geometry, Hirzebruch on characteristic classes. Three days a week a carload of mathematicians, including Michael, Bott and Singer, eagerly traveled to the University to hear these seminars. These were, of course, all topics and contacts which would be influential in Michael's subsequent work. There was another difference from the gentlemanly atmosphere at Cambridge: seminars would abound with comments and criticisms from the audience, probing for more precision and the correct style.

The influence of Serre's Princeton seminar on vector bundles manifested itself in Michael's papers published at the time on the sheaf-theoretical description of characteristic classes and on holomorphic vector bundles on elliptic curves. These were topics he spoke on as he traveled extensively around America, part of the conditions of a Commonwealth Fellowship. He met Chern in Chicago and participated in S. Lefschetz's conference in Mexico.

Back to Cambridge

At the beginning of 1957, Michael returned to Cambridge as a Lecturer, and then, the following year, was also appointed as a Tutorial Fellow at Pembroke College. There, he began to organize colloquia, with professors finding out for the first time what each other was doing. At the same time, he was recounting the ideas he had acquired in the United States to Hodge and others in Cambridge, such as E. C. Zeeman who was actively leading the topology group there. Hodge himself was becoming increasingly occupied with other activities — the Mastership of Pembroke and Secretaryship of the Royal Society — and he handed over to Michael many of the Departmental activities and some of his graduate students too, such as R. Schwarzenberger and I. Porteous.

At this time, in 1957, the Bonn Arbeitstagungen began and Michael became a regular attendee, forging closer links with Hirzebruch. It was a very active period for topology, with J. Milnor's work on manifold classification, Thom's on cobordism theory and Hirzebruch showing the links with algebraic geometry. Although not by training a topologist, Michael began to involve himself more and more in the subject. From a combination of sources — Hirzebruch's integrality theorems, A. Grothendieck's Bonn lectures on the general Riemann–Roch theorem, Bott's periodicity theorem and problems posed by I. M. James, then a colleague in Cambridge — the rudiments of what became K-theory emerged. It became rapidly clear that the K-groups were the correct formal apparatus to solve some very difficult problems in topology.

Michael's work attracted the attention of J. H. C. Whitehead in Oxford, and he went there to give talks, but in 1960 Whitehead died suddenly. The Waynflete Chair which he had occupied was vacant and Michael, at the age of 31, applied for it. In the event, it went to the more senior G. Higman, a group theorist, but a Readership was soon after offered to Michael, and since it provided a relief from college teaching and more time to do research he took it up in 1961.

Oxford

At the time, Oxford was more renowned for its strength in philosophy than in mathematics, but Ioan James had moved there as a Reader, so that topology was certainly represented. Subsequently, Cambridge was to lose Zeeman and others to found the new Mathematics Department in Warwick, and Hodge, still burdened by administration, was happy to send research students over to Oxford to be supervised by Michael, so that the mathematical environment became much more active in the area. The Cambridge students exported to Michael included G. B. Segal, and P. E. Newstead; other students at this time were K. D. Elworthy and B. J. Sanderson. Resources in Oxford for inviting speakers were still slender, so that Michael would spend every sabbatical opportunity visiting Harvard or Princeton, or persuading Bott or Singer to spend their sabbaticals in Oxford. In 1963, the premature death of E. C. Titchmarsh left the Savilian Professorship of Geometry unfilled, and Michael was duly appointed to it.

In the spring of 1962, Singer spent part of his sabbatical in Oxford, and there began the collaboration which culminated in the index theorem in its many forms. The problem originated with the attempt to describe the integrality theorems for characteristic numbers in terms of dimensions of vector spaces. K-theory was a suitable tool to describe the integrality, and Grothendieck's or Hirzebruch's Riemann–Roch theorem gave an answer for the Todd polynomial. During that visit, they rediscovered the Dirac operator, which explained the integrality of the $\hat{A}$-polynomial for a spin manifold. However, it was S. Smale's visit to Oxford on his way back from Moscow which put before them a far more general question — that of finding a formula for the index of a general elliptic operator, a problem of considerable interest to the Russian school and on which Gelfand and his coworkers had made some progress. This wider context stimulated the 18-month pursuit of a proof. In Michael's lecture at the ICM in Stockholm in 1962, the problem and conjectured formulas appeared, but an announcement of the proof was added as a footnote to the published talk.

The period that followed this first successful attack on the Index Theorem led on to a large number of papers exploring generalizations and ramifications of the original idea. Many of these involved group actions, the origins of which lay in walks along dusty paths at Woods Hole near Cape Cod with Raoul Bott. It was a conference in algebraic geometry and number theory, and what started out as a conjecture of G. Shimura on automorphisms of algebraic curves blossomed into a general theory of Lefschetz fixed-point formulas for elliptic complexes, which included H. Weyl's famous character formula as a special case. As well as equivariance, the language of K-theory came to be seen as the appropriate framework for index problems, and in 1968 the first of the five *Annals* papers with Singer gave new proofs of the index theorem in many different contexts, with many different applications. By this time, Michael had been elected a Fellow of the Royal Society in 1962 and won a Fields Medal at the 1966 Moscow ICM for his work on K-theory and the index theorem. He continued to work on indices for a total of over 20 years, as more and more questions needed to be resolved.

The year following the Fields Medal, Michael paid his third visit to the Institute in Princeton, and was approached to see if he would go there as a Permanent Member. L. Hörmander, another Fields Medalist, was just leaving, and after some deliberation back in Oxford he decided to move in 1969.

Princeton again

The Institute held fond memories and, now, new opportunities. There was money to invite people to come and collaborate and no obligation to teach courses — a pure research position for the first time. Moreover, since the Princeton term ended in April, the Atiyahs could return to Oxford where they kept their house, and Michael would participate fully in the life of the Mathematical Institute in Trinity Term (indeed, as I was a graduate student there myself then, I benefited greatly from this arrangement).

In Princeton, the work on the index theorem continued, for families and the mod-2 situation, but Michael soon became aware of a new approach using the heat kernel. Building on the work of Singer and H. McKean, the young V. K. Patodi had found some miraculous cancelations in the asymptotic expansions of the difference of two heat kernels, and produced a proof of the Gauss–Bonnet theorem — the most primitive example of the index theorem. In 1971, Bott and Patodi were invited to the Institute and work began in earnest on this new, more analytical approach to the index theorem; K-theory for the moment lay in the background. With P. Gilkey's use of formal algebra to circumvent Patodi's clever direct manipulations, a new invariant-theory proof of the Index Theorem emerged, one which had an essentially local differential-geometric character. In collaboration with Singer, it was applied to study indices for operators on manifolds with boundary, in particular the signature and Dirac operators, which led to the nonlocal boundary contribution called the η-invariant. One of the guiding motivations in this work was the earlier number-theoretic correction to the signature formula for singularities in Hirzebruch's analysis of Hilbert modular surfaces.

While resolving the problems involved in understanding these new global boundary value problems, Michael made another decision. Despite the heady atmosphere of research in Princeton, like others before and after him, he decided to leave the Institute after three years. He had been offered a Royal Society Research Professorship to return to the UK. It could have been taken up anywhere, but he chose Oxford.

Oxford again

Back in Oxford doing research full-time, and without teaching and administrative duties this time, Michael took on graduate students. In Princeton, apart from G. Lusztig, this did not happen. Moreover, the lack of teaching duties in Oxford meant that he was freer to travel and collaborate. His research began by continuing with topological and geometrical applications of the new Index Theorem, and then he and Singer found an application for a different form of the Index Theorem related to von Neumann algebras. This L^2 index theorem was used for studying infinite coverings of manifolds and unitary representations of Lie groups.

There was another influence in his mathematical work at Oxford during this period, however. At the same time that Michael returned to Oxford, his former colleague as a graduate student in Cambridge, Roger Penrose, came to take up the Rouse Ball Chair in Mathematics. Roger and his students were working out the consequences of his twistor-theoretic approach to the equations of mathematical physics, and while Michael may have been less at home with the physical motivation, they were nevertheless on the same wavelength when it came to the geometry of the Klein quadric, which they had both learned from Todd's book. The first fruit of the interaction was Michael's recognition that the sheaf theory which he had first learnt as a brand-new subject while they were research students together was the appropriate language in which to describe the contour-integral solution of zero rest-mass field equations which Penrose was working on then. This provided a ready-made reservoir of sophisticated techniques to apply to these linear equations. Somewhat later, similar ideas would have even more remarkable repercussions.

In early 1977, Singer paid an extended visit to Oxford and spoke in a series of seminars about what he had learned of the physicists' work on "instantons" — self-dual solutions of the Yang–Mills equations on $\mathbb{S}^4$. His audience was well equipped to understand concepts like principal bundles and curvature, but less about the equations and the role of gauge transformations. Two developments occurred at this time. The first was the work of R. S. Ward, a student of Penrose who had showed using twistor theory that a complex solution of the self-duality equations arose from the data of a complex vector bundle on the projective space $\mathbb{C}P^3$. By chance, Michael had attended the mathematical-physics seminar in which Ward had spoken, and rapidly saw how the Euclidean version of the correspondence worked. The second development was that the Index Theorem could be put to use to actually calculate the dimensions of the moduli space of instantons. The $(8k-3)$-dimensionality of the moduli space coincided with a dimension that W. Barth had calculated for the moduli of certain stable holomorphic bundles on $\mathbb{C}P^3$. This itself was based on a very concrete construction which G. Horrocks had spoken about in Oberwolfach the previous summer. Tying all the threads together, with some differential-geometric vanishing theorems, gave the final outcome in November 1977: a construction of all instantons using just finite-dimensional matrices. At the same time that Michael and I had found this, we heard from Yu. Manin that he and V. G. Drinfeld, who had been following the Oxford developments at a distance in Moscow, had independently derived the same result, which was subsequently known as the ADHM construction of instantons.

This interaction with physics was very influential on the subsequent direction for Michael's mathematics. As it turned out, at the same time that he had been working on the heat-kernel approach to the index theorem, physicists were independently arriving at a form of the same theorem through their study of anomalies. In fact, during an earlier visit to MIT where Michael spoke on the Dirac operator, he had begun to see the role that the index theorem played for physicists. It was during that visit that he first met a young postdoctoral fellow by the name of Edward Witten. By the time of the work on instantons, it was clear to Michael that much more could be gained from both sides by closer ties.

The subsequent research in which Michael engaged for a decade or more was heavily influenced by ideas from physics. This included the application of the Yang–Mills equations to the moduli of stable bundles on Riemann surfaces, on which he worked with Bott. The symplectic aspects of this provided a different viewpoint on moduli spaces in general, an approach taken much further by his student, F. C. Kirwan. It also provided a model for applications of Yang–Mills theory in areas far removed from the original problem of instantons in $\mathbb{S}^4$, ideas taken up in the most spectacular way by his student S. K. Donaldson. The three-dimensional analogue of instantons, the "magnetic monopoles", also occupied his attention.

Perhaps more important than these concrete "classical" interactions of mathematics and physics was Michael's role as both participant and facilitator for the exchange of ideas between the two sides, in particular in discussions with Witten. These included pressing for a quantum-field-theory interpretation of both the Donaldson invariants for 4-manifolds and the Jones polynomials for knots. This was a crucial activity in breaking down barriers of intuition on both sides. By the late 1980s this was a rapidly developing area, so much so that it was said that the award of the 1990 Fields Medals to S. Mori, Drinfeld, V. F. R. Jones and Witten was made to "one classical mathematician and three quantum ones". In 1990, however, Michael's career took another turn.

Cambridge and Edinburgh

The headship of Michael's old college in Cambridge had become vacant, and in 1990 Michael became the Master of Trinity, 40 years after he had entered there as an undergraduate. At the same time, he was persuaded to become President of the Royal Society and, concurrently, the Director of the newly formed Isaac Newton Institute for Mathematical Sciences in Cambridge, whose cause he had promoted strongly. Clearly, with such a volume of administrative activities, mathematical research had to take a back seat.

Michael had already had some experience of public service and learned societies at various stages of his career, having been involved in the Cockcroft Committee for the reform of mathematics teaching in schools, been President of both of the London Mathematical Society and the Mathematical Association, a Vice President of the Royal Society, and active in the International Mathematical Union and in the formation of the European Mathematical Society. Perhaps these activities, as well as his mathematical prowess, contributed to his knighthood, awarded in 1983. In any case, his activities after 1990 won him the Order of Merit in 1992. The Presidency of the Royal Society put him in a position of having an influence on broader science-policy issues, including ethical questions, and his 1995 Farewell Address to the Royal Society provided a much-publicized opportunity to criticise the nuclear-weapons policy of the UK and to deplore the distortions in research which have ensued from the allocation of huge resources to it. He has since carried on this theme through his Presidency of the Pugwash Movement.

In 1997 Michael retired from the Mastership of Trinity, and the Atiyahs went to live in Edinburgh. They had for sometime regularly spent vacation periods in their house in the Cairngorms. Michael is currently active in a variety of international committees, and as an Honorary Professor is also an active presence in the Mathematics Department in Edinburgh.

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