Dear Joan —

Got your letter today — but after our discussions on the phone I found a really easy way to do what I need, and all the proofs were made easier; it was naturally suggested by the ideas you gave me over the phone. I simply define a <u>Heegaard imbedding of a surface</u>, possibly with boundary, (into some 3-manifold $W$) to be any imbedding which is contained <u>in some Heegaard surface of $W$</u>; perhaps this is equivalent to "unknotted surface" in the case that $W = S^3$? In any case, it is the most convenient definition for my purposes, and it gives these results: a) such an imbedding $h$ of a <u>surface with one hole</u> into $S^3$ gives rise to a homomorphism $\rho_h : \mathcal{A} \to Z_2$ (as we discussed on the phone) b) $\rho_{h_1} = \rho_{h_2}$ iff the same spin structure is induced on the surface by $h_1$ and $h_2$; so we can really index the $\rho^{\underline{s}}$ by the spin structures $\xi$ on the surface $M$. c) Any spin structure $\xi$ can be so constructed. A point of interest here is the following: for a surface with one hole we get then $\underline{2^{2g}}$ different $\rho^{\underline{s}}$, not just $2^{g-1}(2^g+1)$; the new ones correspond to imbeddings of $M$ in $S^3$ such that its Arf invariant is 1; the old ones (your previously defined $\rho$'s) come from Arf inv. 0 imbeddings of $M$; these new hom$^{\underline{s}}$ dont arise for closed surfaces; they dont give any new information ~~[illegible]~~ about 3-manifolds either, but just make the theory more "symmetric".

d) Fixing $k \in \mathcal{A}$, we get a function $\sigma_k$ defined on all spin structures $\Phi$: $\Phi$ is an affine space over $Z_2$; for an open surface, $\sigma$ is an isomorphism from $\mathcal{A}/\mathcal{C}$ to the space of Boolean cubic polynomials on $\Phi$ (e.g. dimension $= \sum_{i=0}^{3} \binom{2g}{i}$)

An example of how ~~[illegible]~~ the "new" homomorphisms for an open surface make everything easier: the relations among <u>all</u> the $\rho^{\underline{s}}$ are generated by the following:

$2^3+2^2 < 2^4$

$2^5+2^2 < 2^6$

$2^{2g-1}+2^{g-1} < 2^{2g}$ $\parallel$

TAKE ANY 4-D AFFINE SUBSPACE $R$ OF $\Phi$, ~~[illegible]~~ THEN $\sum_{r \in R} \rho_r = 0$, AND THESE GENERATE ALL REL$^{\underline{S}}$.

I ALREADY HAD SIMILAR THEOREMS FOR A CLOSED SURFACE BUT THESE VERSIONS FOR THE OPEN SURF COMPLETE THE PICTURE IN A SATISFYING WAY; SO I'M WRITING IT UP NOW. IT'S GOING TO BE A LONG PAPER, WITH LOTS OF PREPARATORY MATERIAL. NOW THE KEYSTONE LEMMA ON WHICH IT'S ALL BASED IS THE FOLLOWING: LET $\alpha$ BE A SMOOTH SCC ON $M$ — THEN $\exists$ A NATURAL OBVIOUS TANGENTIAL LIFTING OF $\alpha$ TO A SMOOTH SCC $\vec{\alpha}$ IN THE UNIT TANGENT BUNDLE $FM$ — USE THE TANGENT VECTORS OF $\alpha$. THE KEY LEMMA THEN STATES: IF $\alpha_1, \alpha_2$ ARE SMOOTH SCC'S IN $M$ AND ARE HOMOLOGOUS MOD 2 IN $M$, THEN $\vec{\alpha}_1$ AND $\vec{\alpha}_2$ ARE HOMOLOGOUS MOD 2 IN $FM$. I'VE ENCLOSED MY PROOF OF THE LEMMA. IT IS LONG — TOO LONG. HAVE YOU EVER SEEN SOMETHING LIKE THIS ELSEWHERE? MAYBE I CAN AVOID PROVING THE WHOLE THING WHICH WOULD BE NICER. IF NOT, I WONDER IF YOU SEE A NICE WAY TO PROVE IT. ~~[illegible]~~ ACTUALLY, THE PROOF ITSELF IS NOT TOO BAD OR LONG, BUT ALL THE PREPAROTORY MATERIAL TOO IS WHAT MAKES IT SO. SOMEHOW, I'M SURE THERE IS AN ELEGANT AND SHORT PROOF OF IT; THE PAPER WILL ALREADY BE LONG & TECHNICAL ENOUGH AS IT IS WITHOUT THIS, EVEN GIVEN THAT I'VE CONSIGNED IT TO THE APPENDIX.

I MEANT TO ASK YOU THIS BEFORE: I GOT THE IMPRESSION FROM AN EARLIER LETTER OF YOURS THAT SEIFERT SOLVED THE DOUBLE COSET PROBLEM IN $Sp$ BY MEANS OF HIS VARIOUS LINKING $\#^{\underline{S}}$ AT THE VARIOUS PRIMES. IS THIS SOLUTION OF HIS COMPLETE?, EVEN AT THE PRIME $p = 2$? (I.E. DOES IT HANDLE THE 2-TORSION PART OF THE PROBLEM?) I COULDN'T QUITE DECIDE FROM YOUR LETTER JUST HOW MUCH HE HAD ACTUALLY DONE.

DENNIS.