Dear Joan —

Got your letter — it must have been a nice trip you had, although perhaps not this time of year. Your new work sounds fantastic!

It does indeed seem to be just right for the special Heegaard maps.

I would be amazing if it gave a presentation for $\phi$. I've also got re-interest in the generation of $\phi$ problem and looked at the stuff we did for a while but ran into some problems which seemed tough, so I've abandoned it for now. I've had a different approach which may work to give a minimal set of $W$-type generators (that is, opposite twists on a bounding pair of genus 1). It was going along quite well at first, but just ran into something which is surprising and disturbing. Let $\phi_{1/2}$ be the subgroup of $\phi_1$ (torus with 2 punctures).

$\phi_{1/2}$ is trivial and by a theorem in your book for $\phi_{1/2} \rightarrow \phi_{1}$ is a free group on 2 gens $F_2$. You can show $\phi_{1/2} = \pi_1$ of its commutator subgroup $F_2$

which is not F.G. I'm not sure about $\phi_{1/2}$ for $g \geq 2$, but it seems unlikely that it is not. Now $\phi_{1/2}$ for $g \geq 2$ might be quite different from $n = 0$ or 1, however. Using the same theorem can show the $\phi_1$ is F.G. iff $\phi_0$ is. My idea applies to $\phi_1$, but the $\phi_{1/2}$ example is sufficiently related to make me wonder.

I actually have a candidate set of generators, but to show they work I need to see that certain relations occur in $\phi_{1/2}$, and since this is not F.G. it may not be through. At any rate, the study should be enlightening.

Here, finally, is the paper on torsion of maps in $\phi$ that I haven't written for so long. I decided to extract this material out of the Magnus matrix rep. setting & present it in the way I originally found it — it makes the whole paper much more self contained and easier to read. The torsion along with the "bar calculus"
of my last paper (the one on your homomorphisms) is really a useful tool for suggesting relations in H. I've found a number of such that I would never have found without their help.

I got a letter from M. Kato from Tokyo U. asking for a reprint of "Intersection Invarians on Surfaces." Apparently you mentioned it there, but does he know what an incomplete state it is in? Should I send him the same very rough draft when I see you, or wait until it is written up? It may take some time to do the latter — there are so many other things to do first, it seems, viz:

a) I found a very nice geometric interpretation of the torsion in terms of the Jacobi variety of the surface. In fact, the Jacobi variety gives a new way of looking at maps in H which could be very useful. I will probably work on this next.

b) On looking over your paper on the r-homomorphisms, I was struck by the statement that their seemed to be no "regularity" in the value of \( \mu \mod 8\) for \( \mathbb{Z}_2\)-monodromy spheres. However, using the basic properties of the \( \rho \)'s I can show immediately that \( \mu \mod 4 \) depends only on the symplectic double coset of the gluing map. This easy proof is not computationally effective, however. So I set to work to get an actual formula in terms of the \( \rho \)-matrix itself, and I finally got one, an interesting one involving the self-linking invariants etc. In the process, I found some interesting ways of representing double cosets which might be of use to you in your classification of double coset problem.

Got a letter from Hidden inviting me to the April conference in Hawaii. Are you going? I hope I can make it. Things are tighter here now. Send word on your latest progress —

Dennis