DEAR JOAN

GOT YOUR LETTER & NEW PAPER BUT HAVEN'T HAD TIME TO WRITE. I THOUGHT ABOUT THE IDEA (IN YOUR LETTER) TO GET ~~A~~SYMMETRIC HOMOLOGY SPHERES, BUT I HAVEN'T HAD ANY FURTHER IDEAS. IT SEEMS LIKE A GOOD POSSIBILITY, BUT ONE NEEDS MORE RELATIONS IN $\mathcal{J}$! I HAVE BEEN BUSILY GRINDING OUT SUCH RELATIONS, BUT I SUSPECT OTHERS OF A TYPE WHICH ARE QUITE DIFFERENT FROM THE ONES I HAVE. ALL MINE ARE CARRIED ON GENUS ZERO OR GENUS 1 SUBSURFACES (E.G. THE LANTERN REL IS CARRIED BY A $g=0$ SUBSURFACE) BUT I HAVE NONE THAT ARE ESSENTIALLY GENUS 2 RELATIONS. FURTHERMORE, I HAVE MANY REASONS TO BELIEVE THAT SOME SUCH EXIST, AND THESE WOULD BE MUCH MORE INTERESTING & INSTRUCTIVE THAN THE GENUS 0 OR 1 REL^{S}.

HERE IS MY NEW PAPER! I'M REALLY HAPPY WITH THIS ONE — IT GIVES A VERY CONVENIENT DESCRIPTION OF $\mathcal{J}/\mathcal{C}$, ONE THAT YOU CAN ACTUALLY WORK WITH & DO CALCULATIONS IN! THE BAR NOTATION $(a \to \bar{a})$ IS REALLY EASY TO USE WHEN YOU GET USED TO IT, AND IS A GREAT HELP IN SUGGESTING POSSIBLE RELATIONS IN $\mathcal{J}$, ETC. USING THESE CUBIC POLYNOMIALS FOR EXAMPLE, I CAN PROVE:

1) THE ONLY INVARIANT OF HOMOLOGY SPHERES TO BE DERIVED FROM $\mathcal{J}/\mathcal{C}$ IS THE μ-INVARIANT.

2) ~~#~~ IF, HOWEVER, W IS NOT A $\mathbb{Z}_2$-HOMOLOGY SPHERE THERE ARE NEW INVARIANTS NOT DERIVABLE FROM THE DOUBLE COSETS IN $\mathcal{S}_g$; THIS CONTRADICTS YOUR CONJECTURE IN THE PAPER WITH CRAGGS THAT, FOR

W NOT A $\mathbb{Z}_2$-SPHERE, EQUIVALENCE OVER $\mathcal{S}_p = \mathcal{M}/\mathcal{L} \Rightarrow$ EQUIVALENCE OVER $\mathcal{M}/\mathcal{C}$.

3) IT GIVES A GROUP-THEORETICAL DEFINITION OF μ:

I'M WRITING ANOTHER PAPER NOW SHOWING THAT, IF ω IS THE MOD 2 SELF LINKING FORM OF $M \subset S^3$ AND $\Theta_\omega \subset \mathcal{M}$ IS ~~ALL~~ {all homeomorphisms} HOMEOMS WHICH FIX ω (which fix w) I.E. $\omega(fx) = \omega(x)$ FOR ALL CURVES x ON M, THEN $\mathcal{L}/[\Theta_\omega, \mathcal{L}] \simeq \mathbb{Z}_2$ WITH CANONICAL ISOM GIVEN BY ρ_ω!

I ALSO CONJECTURE THAT IF $\mathcal{M}^{(2)}$ = ALL homeomorphisms HOMEOMS WHICH ACT TRIVIALLY ON H_1 MOD 2, THEN $\mathcal{C} = [\mathcal{M}^{(2)}, \mathcal{L}]$.

BUT: STILL NO WAY TO CALCULATE μ DIRECTLY FROM $f \in \mathcal{L}$! THE ABOVE GROUP-THEORETIC STATEMENT COULD BE USED AS A MEANS TO CALCULATE IT BUT THE ONLY WAY I KNOW TO SHOW AN ELEMENT IS IN $[\Theta_\omega, \mathcal{L}]$ IS BY EITHER a) WRITING IT AS A PRODUCT OF GENERATORS or b) CONSTRUCTING A 4-MANIFOLD. SURELY THERE IS A DIRECT WAY TO DECIDE IF $f \in \mathcal{L}$ IS IN $[\Theta_\omega, \mathcal{L}]$ OR NOT! I'LL BET MAGNUS WOULD HAVE SOME IDEAS ON THIS.

DENNIS