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(1) DEAR JOAN:

I'VE BEEN TRYING TO ORGANIZE MY MATERIAL IN A SUFFICIENTLY READABLE FORM TO SEND TO YOU, BUT IT'S CLEAR NOW THAT IT'S GOING TO TAKE ME SOME TIME, AND IN ANY CASE THE FINISHED PRODUCT WILL APPARENTLY BE RATHER HUGE, & WITH A LOT OF TECHNICAL MACHINERY WHICH MAY PUT OFF A LOT OF PEOPLE; IT WOULD ME, ANYWAY. SO I'VE DECIDED TO JUST SEND YOU A QUICK DESCRIPTION OF IT AND HOPE THAT IT WILL BE OF SOME INTEREST. PERHAPS, AFTER ALL, THERE ARE SIMPLER WAYS OF EXPRESSING ALL THIS STUFF, WITHOUT ALL THE MACHINERY.

SO HERE IS THE BASIC IDEA: I'M WORKING WITH HOMEOM^S OF A SURFACE WITH ONE HOLE, RESTRICTING (VIA ISOTOPY) TO INDUCED AUTOM^S OF A FREE GROUP WITH $2g$ GEN^S (a_i, b_i) WHICH PRESERVE THE ELEMENT $t = \prod_{i=1}^{g} [a_i, b_i]$, WITH APPROPRIATE FURTHER EQUIVALENCES UNDER ISOTOPY; I.E. JUST $\mathcal{M}_{g,1}$ IN YOUR NOTATION, I THINK. (CAN DO IT WITH 0 OR >1 HOLES TOO, BUT THE ALGEBRA GETS MESSIER.) π IS THE FUNDAMENTAL GROUP (FREE ON a_i, b_i) AND $H = \pi/\pi'$ IS H_1. $\mathcal{I} \subset \mathcal{M}$ IS THE SUBGROUP WHICH $= 1$ ON H. IN ORDER TO GAIN INFORMATION ABOUT $\mathcal{I}$, I LOOK AT ITS ACTION ON THE GROUP $A = \pi'/\pi''$. A IS NOT ONLY ABELIAN BUT IS ALSO AN R-MODULE, WITH $R = \mathbb{Z}H$ (THE "ALEXANDER MODULE" OF KNOT THEORY). (IT ALSO HAS ADDITIONAL STRUCTURE, VIZ. AN ANTI-HERMITIAN INTERSECTION PAIRING $A \otimes_R \bar{A} \to R$.

BY TAKING CERTAIN SUBMODULES OF A & QUOTIENTS OF SAME, ALONG WITH THE INDUCED ACTION OF $\mathcal{I}$ ON THESE, WE GET INFORMATION OF INTEREST.

FOR EXAMPLE: SUPPOSE WE REDUCE A TO $A/IA \underset{\text{DEF.}}{=} N_2$, WHERE $I \subset R$ IS THE AUGMENTATION IDEAL (GEN. BY $\alpha - 1$ FOR ALL $\alpha \in H$); THE RESULT IS JUST $N_2 = \pi'/[\pi, \pi']$, A NILPOTENT REDUCTION OF π'.

$\mathcal{I} = \ker(\mathcal{M}_{g,1} \longrightarrow Sp(2g, \mathbb{Z}))$

$\mathcal{J}$ = subgroup of $\mathcal{I}$ generated by twists about separating curves

$f \in \mathcal{J} \longrightarrow f$ has zero "torsion"

(2) $\mathcal{I}/\mathcal{J}$ infinite because $\mathcal{I}/\mathcal{J}$ has a representation onto a free abelian group and $\exists\, f \in \mathcal{I} / \Phi_f \neq 0$

WE THEN FIND THAT THE MAPS OF $\mathcal{I}$ CAN BE REPRESENTED IN A NATURAL WAY BY A MAP $H \longrightarrow N_2$ (OF ABELIAN GROUPS), WITH COMPOSITION IN $\mathcal{I}$ INDUCING ADDITION OF THE CORRESPONDING MAPS $H \longrightarrow N_2$. IN FACT, USING THE FACT THAT N_2 IS NATURALLY ISOMORPHIC TO $\Lambda^2 H$ (EXTERIOR POWER) AND THE SYMPLECTIC DUALITY ON H, WE MAY REPRESENT THIS MAP $H \longrightarrow \Lambda^2 H$ AS AN ELEMENT OF $H \otimes \Lambda^2 H$ IN A NATURAL WAY. FURTHER, IT TURNS OUT THAT THE SET OF SUCH ELEMENTS φ_f ARISING FROM GEOMETRIC MAPS $f \in \mathcal{I}$ IS PRECISELY $\Lambda^3 H \subset H \otimes \Lambda^2 H$ WHEN $g > 2$, AND THAT THE ASSIGNMENT

$$\mathcal{I} \xrightarrow{\varphi} \Lambda^3 H$$

IS A HOMOMORPHISM. I'VE BEEN CALLING φ_f THE TORSION OF f BECAUSE OF ANALOGIES WITH DIFFERENTIAL GEOMETRY. THE TORSION OF A MAP $f \in \mathcal{I}$ IS INTERESTING FOR THE FOLLOWING REASONS:

a) ANY MAP WHICH IS (ISOTOPIC TO) A PRODUCT OF TWISTS ON BOUNDING LOOPS (LOOP = IMBEDDED S^1) HAS ZERO TORSION; SO TORSION-FREE IS A NECESSARY CONDITION FOR f TO BE IN $\mathcal{J}$, THE SUBGROUP OF $\mathcal{I}$ GENERATED BY SUCH TWISTS. FOR EXAMPLE:

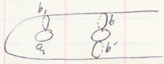
b) SINCE Λ^3 IS A FREE ABELIAN GROUP, IF $\varphi_f \neq 0$, THEN $\varphi_{f^n} = n\varphi_f \neq 0$, I.E. f^n IS NOT IN $\mathcal{J}$ EITHER, FOR ALL n, AND SO $\mathcal{I}/\mathcal{J}$ IS NOT FINITE (ANSWERS YOUR QUESTION TO KIRBY IF WE FIND EVEN ONE f WITH $\varphi_f \neq 0$, FOR WHICH SEE BELOW; THIS STILL WORKS FOR $\mathcal{M}_{g,0}$, BUT AS I SAID, IS MESSIER ALGEBRAICALLY).

c) CHILLINGSWORTH ASKED: IF $f \in \mathcal{I}$ AND PRESERVES ALL "WINDING NUMBERS", IS IT IN $\mathcal{J}$? ANSWER IS NO, FOR THE FOLLOWING REASON:

THE ACTION OF f ON THE WINDING NUMBERS CAN BE REPRESENTED BY A HOMOLOGY CLASS $\psi_f \in H$, AND WE FIND THERE IS A NATURAL HOMOMORPHISM

$\Lambda^3 H \to H$ TAKING φ_f TO ψ_f; SO IF $\varphi_f \neq 0$ IS IN THE KERNEL OF THIS MAP (ALWAYS NON TRIVIAL FOR $g>2$) THEN f PRESERVES WINDING NUMBERS BUT IS NOT IN $\mathcal{I}$.

AS AN EXAMPLE OF WHAT THE TORSION LOOKS LIKE, LET $f = T_{b'} T_b^{-1}$:



WITH $b \simeq b' \simeq b_2$

THEN WE GET $\varphi_f = a_1 \wedge b_1 \wedge b_2$ IN $\Lambda^3 H$.

$\mathcal{I}$ IS GENERATED BY 1) TWISTS ON BOUNDING LOOPS AND 2) THE ABOVE TYPE f. NOW IF γ & γ' ARE BOUNDING LOOPS, THEY ARE EQUIVALENT OVER $\mathfrak{M}$ (I.E. T_γ & $T_{\gamma'}$ ARE CONJUGATE OVER $\mathfrak{M}$) IFF THE TWO CURVES SEPARATE THE SURFACE INTO PIECES OF THE SAME GENERA. BUT WHEN ARE THEY EQUIVALENT OVER $\mathcal{I}$? IT IS NOT TOO HARD TO SEE THE FOLLOWING: γ (& γ') REPRESENTS A WELL-DEFINED ELEMENT IN $N_2 = \Lambda^2 H$, AND γ & γ' ARE EQUIVALENT OVER $\mathcal{I}$ IFF THEY GIVE THE SAME ELEMENT IN N_2.

TO GET CONJUGACY EQUIVALENCE OF TYPE 2 GENERATORS WE HAVE:

d) SUPPOSE (b, b') ARE TWO DISJOINT HOMOLOGOUS LOOPS & f THE CORRESPONDING TYPE 2 MAP; LIKEWISE (c, c') & g. THEN f & g ARE CONJUGATE OVER $\mathcal{I}$ (OR EQUIVALENTLY, THE PAIR OF CURVES (b, b') ARE EQUIVALENT TO (c, c') OVER $\mathcal{I}$) IFF $\varphi_f = \varphi_g$.

OF COURSE, THIS DOESN'T PROVE THAT $\mathcal{I}$ IS INFINITELY GENERATED, BUT MAY BE APPLICABLE TO YOUR WORK ON THE HOM'S $\mathcal{I} \to \mathbb{Z}_2$. NOTICE THAT ANY ATTEMPT TO PROVE THAT $\mathcal{I}$ IS INFINITELY GENERATED VIA ITS HOM'S INTO ABELIAN GROUPS MUST ASSUME THAT $\mathcal{I}/\mathcal{I}'$ IS INFINITELY GENERATED. FOR CERTAIN VAGUE REASONS I FEEL THAT, QUITE POSSIBLY, $\mathcal{I}/\mathcal{I}'$ IS FINITELY GENERATED, IN WHICH CASE THIS APPROACH WOULD NOT WORK. I ALSO HAVE SOME (ALSO VAGUE) IDEAS FOR PROVING $\mathcal{I}$ INF. GENERATED BY LOOKING AT ITS ACTION ON THE MODULE A — BUT NOTHING REAL YET.

NOTE THAT SINCE φ IS ACTUALLY A REPRESENTATION OF $\mathcal{J}/\mathcal{J}'$ ONTO $\Lambda^3 H$, WE MIGHT EVEN CONJECTURE THAT $\mathcal{J}/\mathcal{J}' \simeq \Lambda^3 H$. THAT THIS IS NOT TRUE, I JUSTIFY IN THE FOLLOWING WAY: YOUR $\mathbb{Z}_2$ HOM^S "SEE" THE ROCHLIN INVARIANT, WHEREAS THE TORSION DOES NOT, BY VIRTUE OF:

e) IF $f \in \mathcal{J}$ DEFINES A HOMOLOGY SPHERE (VIA HEEGARD DECOMP.) THEN WE CAN REDUCE f (VIA HANDLEBODY CHANGES) TO A TORSION FREE MAP g; THAT IS, EVERY HOMOLOGY SPHERE CAN BE DEFINED BY A TORSION FREE HEEGARD MAP.

IN OTHER WORDS, WE CAN'T GET INVARIANTS OF HOMOLOGY SPHERES FROM THE TORSION. THIS LEADS ME TO THE NEXT PART: A WAY OF BOOTSTRAPPING UP BIGGER ~~REP~~ ABELIAN REPRESENTATIONS OF $\mathcal{J}$, AND ALSO POSSIBLY GETTING THE ROCHLIN (OR OTHER) INVARIANTS. WE LOOK NOW AT THE TORSION-FREE MAPS ($\mathcal{J}_1 \subset \mathcal{J}_0 = \mathcal{J}$ I'M CALLING IT: IT'S NORMAL IN $\mathcal{M}$).

$f \in \mathcal{J}_1$ INDUCES NOW A MAP $H \longrightarrow IA/I^2A \underset{\text{DEF.}}{=} N_3$; SUCH MAPS CAN BE REPRESENTED BY A TENSOR OF RANK 4 IN H.

THIS TENSOR HAS TWO PARTS (IT IS A DIRECT SUM) BUT I'LL CONCENTRATE ON JUST THE PART I KNOW THE MOST ABOUT NOW: IT IS A TENSOR R_{ijkl} WHICH I'M CALLING THE CURVATURE OF f BECAUSE R SATISFIES ALL THE SYMMETRIES (+ JACOBI RELATIONS) OF THE CLASSICAL RIEMANNIAN CURVATURE TENSOR. I DON'T KNOW EXACTLY WHICH R^S ARISE FROM GEOMETRIC MAPS, BUT DO KNOW THAT TENSORED WITH $\mathbb{Z}(\frac{1}{2})$ (DYADIC RATIONALS) IT IS EVERYTHING, AND THAT IN THIS TENSORED SPACE, WE CAN REDUCE R TO ZERO BY HANDLEBODY CHANGES; THERE IS STILL A POSSIBILITY THAT THERE ARE $\mathbb{Z}_2$ INVARIANTS **LURK**ING IN THERE. IF SO, IT WOULD BE INTERESTING TO SEE IF THESE ARE RELATED TO YOUR OWN $\mathbb{Z}_2$ MAPS. COULD YOU SEND ME WHAT YOU HAVE AVAILABLE ON THAT WORK?

I'M DOING SOME GIGANTIC CALCULATIONS TOWARDS CLEARING UP THESE QUESTIONS (ANOTHER REASON WHY I MAY NOT GET AROUND TO ACTUALLY WRITING FOR A WHILE) AND I'LL TRY TO KEEP YOU INFORMED.

DENNIS JOHNSON

YOU CAN ALSO SEND MAIL TO ME AT WORK:

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