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Dear Zhang,

Thank you for your letter. I am happy to hear that you are thinking about the heights of Heegner cycles, for modular forms of weights $2k \geqslant 4$. As far as I know, the paper of Brylinski in Duke Math J. is the only work that has been done on this. I think he uses Deligne's proposed "cycle" in the local system $Sym^{2k-2}(H^1(E))$ of motives on the modular curve X, where E is the universal elliptic curve

Indeed, the class $v_x = (\text{mult by } \sqrt{-D} \text{ on } E_x)$ gives an endomorphism of trace 0, so a class in $Sym^2(H^1(E))(1)$. Then $v_x^{(k-1)}$ gives a class in $Sym^{2k-2}(H^1(E))(k-1)$ using the map

$$Sym^{k-1}(Sym^2(\mathbb{C}^2)) \longrightarrow Sym^{2k-2}(\mathbb{C}^2)$$

given by projection to the highest weight piece. My student, Rhonda Hatcher, used similar classes to study the values (not derivatives) of Rankin L-series: Canadian J. Math, (1990), Vol 42, especially page 545.

You ask about subtracting a term like v_0 corresponding

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to the cusp 0, but this is <u>not</u> <u>necessary</u> once the weight is $2k \geq 4$. For there is non-trivial monodromy which forces $H^2(X, Sym^{2k-2}) = 0$, so any cycle is automatically homologically trivial (unlike the case when $2k=2$, $Sym^{2k-2} = \mathbb{Q}$)

The problem with the approach of Brylinski is that the height pairing is <u>only</u> defined as a sum of local terms, and has no global interpretation. It's not even clear what subgroup of the "cycles" should be its radical. I hope you can make some progress on this.

Best wishes,

[illegible]