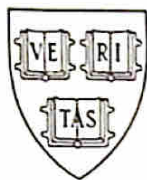


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January 3, 1995

Dear Zhang,

Thank you for your letter. I am happy to hear that you are thinking about the heights of Heegner cycles, for modular forms of weights  $2k \geq 4$ . As far as I know, the paper of Brylinski in Duke Math J. is the only work that has been done on this. I think he uses Deligne's proposed "cycle" in the local system  $\text{Sym}^{2k-2}(H^1(E))$  of motives on the modular curve  $X$ , where  $E$  is the universal elliptic curve

Indeed, the class  $v_x = (\text{mult by } \sqrt{-D} \text{ on } E_x)$  gives an endomorphism of trace 0, so a class in  $\text{Sym}^2(H^1(E))(1)$ . Then  $v_x^{(k-1)}$  gives a class in  $\text{Sym}^{2k-2}(H^1(E))(k-1)$  using the map

$$\text{Sym}^{k-1}(\text{Sym}^2(\mathbb{C}^2)) \rightarrow \text{Sym}^{2k-2}(\mathbb{C}^2)$$

given by projection to the highest weight piece. My student, Rhonda Hatcher, used similar classes to study the values (not derivatives) of Rankin L-series: Canadian J. Math, (1990), vol ~~41~~ 42, especially page 545.

You ask about subtracting a term like  $v_0$  corresponding

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to the cusp 0, but this is not necessary once the weight is  $2k \geq 4$ . For there is non-trivial monodromy which forces  $H^2(X, \text{Sym}^{2k-2}) = 0$ , so any cycle is automatically homologically trivial (unlike the case when  ~~$2k=2$~~ ,  $\text{Sym}^{2k-2} = \mathbb{Q}$ )

The problem with the approach of Brylinski is that the height pairing is only defined as a sum of local terms, and has no global interpretation. It's not even clear what subgroup of the "cycles" should be its radical. I hope you can make some progress on this.

Best wishes,

Rudolf H.