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To: szhang@math
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January 4, 1995

Dear Zhang,

I sent a reply to your previous question by ordinary mail. As for the one you just asked, I'm afraid there is no good reference in the literature. It's my way of phrasing results of Shimura, Doi-Naganuma, and Cherednik-Drinfeld. What Shimura proved (see chapter 9 of his book Arithmetic Theory of Automorphic Functions) was that associated to a quaternion algebra R over a totally real field k, which is split at precisely one real place v, one can define an algebraic curve over k which is uniformized by the Fuchsian group associated to R at the completion k_v.

The Doi-Naganuma, in an Annals paper, proved that over the other real places w of k, this curve was uniformized by a Fuchsian group associated to the quaternion algebra R', which is split at w, ramified at v, and otherwise locally isomorphic to R. Finally, Cherednik and Drinfeld proved that the same was true (using a p-adic uniformization) at the places w ramified in R.

My point in rephrasing these results is that the curve is canonically associated to the odd set of places Sigma = v + ramified places in R. If one wants a uniformiztion at any of these places, one uses the quaternion algebra ramified at Sigma - that place. This point of view was a big help to me in the calculation of local heights. Also, the supersingular points (mod p) for a place not in Sigma are indedxed by the ideal classes of the definite quaternion algebra ramified at Sigma + p. So the local height at any place of k is calculated using a quaternion algebra at distance 1 from the odd set Sigma.

I hope this helps.

Dick Gross