$A_\varphi = \langle \pm 1 \rangle^t \quad t = \#$ of V_i with $\operatorname{Ind} V_i =$ sym / orth mod [n-w?]

$a = a_1 \times a_2$ in $A_{\varphi_1} \times A_{\varphi_2}$.

$\varepsilon(a) =$ local sign of $\operatorname{Ind}(V_i^{a_1} \otimes W_i^{a_2})$ sympl.

Global $\quad \pi = \otimes \pi_v \quad U_n$

$\pi' = \otimes \pi_v' \quad U_{n-1}$

assume $\operatorname{Hom}_{U_{n-1}}(\pi_v \otimes \pi_v', \mathbb{C}) = \mathbb{C}$

$L(\operatorname{Ind}(V \otimes W), s)$ is non-zero at $s = \frac{1}{2}$

$\quad \parallel$

$L_E(V_n \otimes W_{n-1}, s)$

$\updownarrow$

$\int_{U_{n-1}(\mathbb{A})} f \otimes f' \neq 0$

$${}^LG \xrightarrow{ad} GL(\hat{\mathfrak{g}})$$

$$\xrightarrow{??} GL(V)$$

symplectic

$$\frac{L(V \otimes W, \frac{1}{2})}{L(ad, 1)} = \langle f, f \rangle$$

$SO_{2n+1} \times SO_{2n}$

$L(V_{2n} \otimes W_{2n}, \frac{1}{2})$

dim $4n^2$

$L(ad, 1)$

$\binom{2n+1}{2} + \binom{2n}{2} = 4n^2$

dim $V \otimes W$ = dim ad $\quad *$

Shimura var $U_{n-1,1}$

$U_{n-2,1}$

$L'(\frac{1}{2})$?

$SO_{2,2n-2}$

$SO_{2,2n-1}$

Gross-Prasad.

E/k quad. $e: Gal(E/k) \to \langle \pm 1 \rangle$

$G = U_n(E/k)$

What is a Langlands param for G

$^L G = \hat{G} \rtimes Gal(E/k) = GL(V) \rtimes Gal(E/k)$ $\dim V = n$

$\varphi : W_k' \longrightarrow {}^L G$

$\triangledown \qquad \triangledown$

$\varphi_E : W_E' \longrightarrow GL(V)$

When does a repr φ_E extend to a parameter?

$Ind_E^k V$ has dim $2n$ $\Longleftrightarrow$ symplectic n even
orthog of det e n odd

$V^* \simeq V^\sigma$

$U_{n-1} \hookrightarrow G = U_n \times U_{n-1}$ $W_n \supset W_{n-1}$.

$^L G = (GL(V) \times GL(W)) \rtimes Gal(E/k)$

$\longrightarrow Ind_E^k (V \otimes W)$ dim $2(n)(n-1)$ natural symplectic

dim $\approx \dim(\mathfrak{g})$

$\{\pi_n\}$ of $U_n(k)$ $\sum_{L\text{-pack}} Hom_{U_{n-1}}(\pi_n \otimes \pi_{n-1}, \mathbb{C})$ has dim 1

$\{\pi_{n-1}\}$ $U_{n-1}(k)$

$V \simeq \oplus\, n_i V_i$ $\underline{V_i \simeq \bar{V}_i^*}$ or $V_i \simeq \bar{V}_j^*$

$W = \oplus\, n_i W_i$ c_i