CELEBRATIO MATHEMATICA

Saunders Mac Lane

JOHN G. THOMPSON

THE MAC LANE LECTURE

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Shortly after the death of Saunders Mac Lane in April, Krishna [Alladi] asked me if I would be willing to speak publicly about Saunders. I agreed to do so, but asked for time to think about and to prepare my remarks. In the meantime, Saunders's autobiography [2005] has appeared, and it has been helpful to me.

I expect that everyone here is aware of the book and the movie "A beautiful mind" which explore the life of John Nash. You will know that for many years, Nash was insane with schizophrenia. For most of us, and certainly for me, insanity is frightening and far from beautiful. I submit that Saunders had a genuinely beautiful mind. Except for an elite few of us, Mac Lane's life and work do not have the drama and punch of Nash's odyssey. I see my note today as a recorder, neither a hagiographer nor a debunker.

Mac Lane's mental world had great lucidity and covered much territory. He took mathematics seriously and he was unswerving in the clarity with which he probed the structures and concepts that support logical thought. I shall try to give some indications of why I make such an assertion. His autobiography makes a good case that my feelings about his mind are well founded. There are intellectual and psychological nuggets in his account which merit examination.

I cannot recall the transition moment when I began to think of him as Saunders and address him by his first name. I would not have thought to address him as Saunders during the time he was my thesis advisor. We met at my home around my fortieth birthday, and by that time it came easily to me to call him Saunders. In different periods and places, the progression of an ongoing teacher-student pairing takes different paths from intensity to estrangement. With Saunders and me, our mathematical paths crossed only briefly, but very intensely. Even though we diverged professionally, we saw each other socially though the years, and there still remained in my mind a sense of being sheltered by him, which I think is one of the gifts that a good father provides to his offspring. I interject here that my own father also provided that sheltering sense to me.

Several obituaries of Saunders appeared shortly after his death, and several mathematicians were quoted. Here is a sampling:

> He was one of the most important figures in the University of Chicago Mathematics Department, or indeed in American mathematics.
>
> . . . extremely energetic, dynamic, clear-headed, opinionated, a raconteur.
>
> With Sammy Eilenberg he created a new way of thinking about mathematics
>
> Category theory is still exploding in its influence after 60 years.

> Mac Lane was one of the pioneers of algebraic topology.
>
> He wrote famous texts.
>
> He has left a unique body of material for future historians.

These snippets have been culled from comments of Eisenbud, May, Lawvere, and Johnstone. Keeping in mind that, when person A makes a comment about person B, we may learn something about both A and B, these quotes carry quite a bit of information.

Saunders was born in 1909, the eldest of his parents' four children. His paternal grandfather became a Presbyterian minister in Ohio, who resigned his position there after being charged with heresy for preaching evolution. He took up a position as minister in a Congregational church in New Haven, Connecticut. Saunders's father also was a minister. The father falling ill with tuberculosis and possibly the after-effects of the 1917 influenza epidemic, it was decided that Saunders, his brother and his mother take up residence with the grandfather in Leominster, Massachusetts, while his father tried to recuperate in a sanitarium. Concerning this period in his life, Saunders wrote:

> During my high-school years, the Ku Klux Klan was active and apparently critical of Catholic doctrines. A Congregational church downtown received and praised Klan members with these views, and the excessive attacks on Catholicism displeased my grandfather. He proceeded to open his church on a weekday to give a two-hour lecture on the origins of the Protestant/Catholic divide, which was well attended and impressively done. I admired his devotion and vigor in defense of tolerance, and marveled at his decisive activity at nearly 80 years of age.

In this passage, Saunders achieves at least two goals. He bows to the insight and sensitivities of his remarkable grandfather, and he establishes that he was a teenager on the *qui vive* intellectually. He continues and qualifies his admiration of his grandfather's lectures:

> This does not mean, however, that I understood his ideas. I did join his church, and in the process of doing so, remember being questioned carefully by an elderly member about my beliefs. I answered seriously, but at the same time, kept some reservation about certain points of doctrine. I struggled with aspects of my ministerial heritage, but I did not often approach my grandfather for advice and wisdom. On one occasion, I asked him about the purpose of individual life. He responded that we were there to exhibit the glory of God; this conclusion stopped me cold — God's glory was not visible to me.

This passage is the only place in the autobiography which mentions Saunders's religious beliefs. His grandfather's life was roiled occasionally, and Saunders's reticence is an understandable reaction. Given his grandfather's putative heresy, Saunders may have taken the path so poignantly traced out in Matthew Arnold's "Dover Beach."

Saunders's father died in early 1924, when Saunders was 14. His sister died at the age of 4, when Saunders was 7. These deaths contributed, I think, to the seriousness with which Saunders faced life.

All five of Saunders's uncles went to Yale, as did both of his grandfathers, so it was natural for Saunders to go to Yale too. He includes in his autobiography a photograph of himself around the time of his graduation. This photograph appeared in the (Norwalk) *Times* on Sunday, March 31, 1929, with the caption: "Attains highest academic standing in the history of Yale. Saunders Mac Lane of Norwalk, who maintained an Average of 384.8 in his studies for the first two and a half years at the University, for which he was honored by being elected President of the New Haven Chapter of Phi Beta Kappa."

Saunders closes his account of the social aspects of his Yale years:

> Emphasis on tradition was still powerful at Yale, ... The famous Tap Day came at the end of junior year. All of the juniors congregated in the old campus hoping to be selected while the senior members of the secret societies walked among the juniors tapping selected candidates and saying "Go to your room." I stood there with all my classmates. As a junior Phi Theta Kappa I must have considered I was worth a tap, but it never arrived. I wept no tears — I had aimed my undergraduate career in the appropriate direction. I cared more for lessons than for student customs. As a sophomore, I declined to join a fraternity; as a senior, I would not have made a loyal member of a secret society. Make no bones about it, any education involves choices, and a college student must choose.

While Saunders was dismissive of Tap Day at Yale, he was not at all dismissive of the course work during his undergraduate years, and his high grade point average reflects his studiousness. His interest in physics never left him, but he found himself drawn to mathematics more strongly, not however to the exclusion of other intellectual interests.

The Saunders Mac Lane whom I came to know fledged at Yale, as the following statement shows:

> It seemed as if all of our attention was directed toward knowledge that was already known; therefore, during the first years of my undergraduate education, I put my own emphasis on acquiring universal knowledge — the assimilation and organization of everything known.

Had I come across references to "universal knowledge" and "everything known" and not had the benefit of knowing Saunders, I would have written him off as some sort of intellectual Walter Mitty. Since I have some idea of the power of his mind, however, I think that a more realistic comparison would be to Leibnitz. Leibnitz's monads and Mac Lane's categories may be viewed as attempts to reach some meaningful bedrock. For me however, universal knowledge is only approachable asymptotically, and convergence is very slow.

Certain intellects strive to explore, unite, and most importantly, to understand new ideas. Saunders was of this stripe. As a special case, he was excited by the lectures of Øystein Ore at Yale, who gave an account of Galois theory and groups. Ore had recently studied at Emmy Noether's school of abstract algebra in Göttingen, and Saunders found himself attracted to Germany.

I interject here that Professor Ore was sympathetic to me when I went to him as a freshman with a note I had written about prime numbers. With his advice, this note was published in the *American Math Monthly* when I was a sophomore.

Saunders decided to go to the University of Chicago for graduate work, at least partly because Hutchins urged him to do so. I find it fascinating, even thrilling, to note just how much intellectual growth Saunders maintained for decades. But he did not completely exclude the softer arts during his Chicago graduate days:

> I enjoyed various diversions at Chicago: I learned to play bridge..., I visited the Lyric Opera, but found Wagner's Rhine maidens too buxom; and my friend Manson Benedict and I invented a game of three-dimensional chess. In the spring of 1931, I went along with Manson Benedict and his date Dorothy Jones, a graduate student in economics from Arkansas, to search for a communist meeting in downtown Chicago led by the survivors of the Haymarket Massacre of the late 1880s. We failed to find it, but I later dated Dorothy, as will appear.

His first year of graduate work at Chicago was a let down:

> Overall, I found this year of graduate work at Chicago disappointing, especially because I could not see any possibility for a Ph.D. thesis on logic. I was disappointed with other aspects of mathematics at Chicago as well.... Perhaps I was confirming Professor Ore's judgment on mathematics. At any rate, I wrote the Institute for International Education, applied for a fellowship to study in Germany and won an award to do so....
>
> My shift from Chicago to Göttingen had a strong, positive, motivation: I wanted to write a thesis on logic.

Saunders did indeed write an accepted thesis on logic in Göttingen. He and Dorothy Jones were also married in Germany. Both his marriage and his interest in logic survived for over 50 years. Noether, Hilbert, Weyl and others introduced Saunders to the vitality of German mathematics, a vitality which was decapitated abruptly when Hitler took over the Reich.

Saunders and Dorothy returned to the United States in 1933, and Saunders pursued postdoctoral work at Yale for a year, and then went to Harvard as a Pierce instructor. He taught a course using van der Waerden's text "Modern algebra."

In 1936, a result of Saunders was published in a journal. This result gives a set of generators for the fundamental group of a planar graph. He notes that he learned some topology from Whitney and some algebraic geometry with Walker. After stints at Cornell and Chicago, Saunders returned to Harvard and with Garrett Birkhoff wrote and published "Survey of modern algebra," a text with considerable influence over several decades. There is a brief remark by Stendhal which to my mind illuminates one of the virtues of the Birkhoff–MacLane textbook.

In Stendhal's autobiography "The life of Henri Brulard," Stendhal laments that he was unable to understand why the product of two negative numbers is a positive number, and he lambasts one of his teachers for spouting some nonsense about this "fact." Once one has the axioms for a ring, the equality $(-a)(-b) = ab$ becomes an exercise, an exercise which appears explicitly in Birkhoff–MacLane. I mention this since the point which troubled Stendhal also troubled me to the verge of tears in junior high school, and was not resolved until I read Birkhoff–MacLane.

I have mentioned tears and I record that, during one of my weekly meetings with Mac Lane while writing my thesis, I broke down in sobs. The precipitating cause was no doubt a combination of my awareness that I was not close to where I wanted to be, proof-wise, and Mac Lane's all too clear recognition of my vagueness. My memory is that he was shocked to witness tears elicited by his straightforward remarks.

During World War II, Saunders and other mathematicians were recruited to aid in the war effort. Among other problems with a mathematical component, they examined how to shoot down aircraft. Saunders mentions the pursuit curve, which in two dimensions is the curve traced by a point in the plane such that the tangent at each point is directed toward a second point in the plane which is moving with constant velocity in a straight line. Similar problems of fire control are mentioned by Norbert Wiener in his book on cybernetics.

Saunders was somewhat dismissive of the degree to which documents were classified as secret by the government, and could see no value in classifying trigonometric tables. He states that the highest classification was "Burn after reading," and he jokes that there may have been a yet higher level, "Burn before reading." This anecdote and others indicate that Saunders was not bowled over by authority.

It is instructive to read Saunders's attitude toward joint work.

> Traditionally, most mathematicians conducted research by working alone, as in the case of Gauss and Poincaré and many others, eminent or not. . . .
>
> In my own case, I came to realize that my research was hampered by a considerable lack of broad knowledge of mathematics.

It was somewhat of a revelation to me to learn that Saunders on several occasions actively sought to collaborate with others. Anyone with even a passing acquaintance with Saunders, however, will know of the fruitful and enduring collaboration between Eilenberg and Mac Lane, which led to Eilenberg–MacLane spaces, and even more importantly to category theory. Saunders discusses several aspects of category theory, starting with the "well known map θ sending each space V to its double dual V^{**}." As the earlier quote of Lawvere that category theory is still "exploding" emphasizes, MacLane's influence is widely appreciated. Saunders describes how Sammy Eilenberg dumped Lawvere's thesis literally in his lap during a flight that they shared, and how Lawvere eventually convinced Saunders that it is possible, perhaps advisable, to eliminate the notion $a \in S$ from the foundations of set theory. Set theory without elements exists and, if I have it right, leads to topos theory.

I want to bring in President Clinton's big book "My life," which I read last summer rather carefully. The number of people named in this book is prodigious. Take me as a babe in the woods, but President Clinton is a stellar networker. Perhaps that is in the nature of politics. There is a reservation in me, indicative of the more closed world in which I have moved, that there is a reprehensibility in that excessive outreach. Saunders would not have agreed with me, for he was not only an active collaborator, he was also a good networker, to the benefit of mathematics. His enthusiasm for good ideas and the people who have them was marked, and he did not hesitate to express himself in lectures and papers.

Saunders admitted that he may have been too zealous in promoting reform in the teaching of mathematics, and in his autobiography he mocks himself by mentioning the kindergarten child who is taught that a set consisting of exactly the two elements a and b might be symbolized as $\{a, b\}$. Johnny's parents ask the kindergarten teacher how he is doing, and she tells them, "Yes, he understands sets, but he has trouble writing the curly brackets." I do not enter the contentious arena concerning the age at which a child can or should be introduced to set theory, but Saunders was intrepid and not at all a crackpot. He puts in a good word for the two-column method of teaching Euclidean geometry, but he was also convinced of the importance of abstract reasoning.

Saunders was aware of his Scottish heritage, and he dressed accordingly, trotting out the relevant tartan from time to time. In accordance with the stereotype of the parsimonious Scot, Saunders was tight-fisted about money. I cannot remember the circumstances, but on one occasion he remarked that money is a module over the integers.

On the few social occasions that I attended at his apartment in Chicago or at the dunes, he did not serve alcohol and, as far as I can tell, alcohol played no meaningful role in his life. I suspect that he had enough experience of drinking to think that his mind was more interesting when sober than tipsy, a view which many would question about themselves. With his well-furnished mind, it was surely true of Saunders.

Saunders enjoyed doggerel, and he sprinkled it throughout his autobiography. At math parties, he gave renditions of the ditty about the zeta function, "Where are the zeroes of zeta of s?" His enthusiasm was contagious.

My graduate years at Chicago brought me in contact with Marshall Stone who played such an important role in building a world-class mathematics department. Saunders was a part of this stellar constellation and, in his autobiography, he gives an entire chapter to this period. I can do no better than to read it to you.

> By good instincts and foresight, Marshall Stone succeeded in creating an innovative and inspiring new department of mathematics at Chicago, devised by a happy combination of an international senior faculty, an ambitious junior faculty, and an unusually lively group of graduate students. For a period, it was arguably the best department in the world.
>
> The senior faculty members were Adrian Albert, S. S. Chern, Marshall Stone, André Weil, Antoni Zygmund and myself. Albert was a holdover from the old Chicago department, but Stone brought in Chern, me, Weil, and Zygmund, a remarkable quartet of senior appointments in such a short time that would have given a big shot in the arm to any department, and it certainly did so to Chicago.
>
> Albert had started as a student of Leonard Dickson in algebra; he was aware of the nature and troubles of the old Bliss department and was happy to join Stone in the new direction. His research interests continued some of Dickson's in group theory, linear algebras, and nonassociative algebras.
>
> Chern, from China, had studied in Europe with Blaschke and Élie Cartan. He knew and understood the strength of Cartan's use of differential forms in geometry.
>
> Stone, as already noted, led in the use of Hilbert-space methods in analysis and physics. The new department was definitely his design — he was, in effect, a dictator, not just a chairman.
>
> Weil had grown up in the demanding and high-reaching world of French mathematics. He found decisive results in algebraic geometry, and he and his young colleagues in France had realized

the importance of the abstract methods newly used in Göttingen. They started the influential Bourbaki group in the ambitious project of an exposition of all of basic mathematics.

Zygmund and his wife had escaped Poland at the start of the war. Between the wars, Polish mathematicians established a lively school of mathematics, deliberately restricting their research interests to chosen topics in topology, logic, and analysis. Zygmund was a devoted specialist in analysis, particularly in harmonic analysis. He was equally devoted to his many students. Other refugee mathematicians from Poland displayed a similar devotion to their own specialties.

Stone's junior faculty, who all began as assistant professors, consisted of Paul Halmos, Irving Kaplansky, Irving Segal and Edwin Spanier. Halmos, a Hungarian, was a protégé of von Neumann, and an enthusiastic expositor on vector space theory. Kaplansky, my first Ph.D. student, had met Albert during the war, and actually had come to Chicago on Albert's initiative a year before Stone arrived. Segal, another von Neumann protégé, studied at Princeton and taught briefly at Harvard. Spanier, a recent Princeton student in topology, had learned the flourishing new ideas of algebraic topology from Steenrod.

At the start of the Stone Age, Otto Schilling was still in Chicago; his mathematical interests in algebra had been interrupted by legal problems involved in a troubled financial inheritance from Germany — my collaboration with him sadly came to an end. Eventually, he moved to Purdue University. A few years later, I. N. Herstein, an algebraist, joined the department, and several of the younger mathematicians from the Bliss–Lane department left. Professor Lawrence Graves retired, as did H. S. Everett, who had effectively managed correspondence courses in mathematics.

With the start of the Stone department, many new, able graduate students arrived in Chicago. A number of them were supported by the G.I. Bill, which provided adequate stipends, and many students came because they had used the Chicago correspondence courses prepared by Professor Everett during their war service. Other students in Urbana or New York City heard of exciting things going on in Chicago, and so came to study there, and there were also several fellows who came from Europe to visit Chicago. One British visitor returned to England to build up a Chicago-style department in Warwick.

As chairman, Stone was the leader and manager; he was forceful and judicious — perhaps growing up as a son of a chief justice of the Supreme Court helped. The story goes that at one point, he was busy in his offices while a student, Bert Kostant, waited patiently at the door. Professor Weil came to see Stone on some matter, opened the office door, and walked in, whereupon Stone pointed out that the student had been there first.

Stone wholly reorganized the graduate program in mathematics; previously, the final Ph.D. examination formally covered all 27 graduate courses the student had taken. I recall one example where a student had taken all 27 courses, and finally, in absentia, finished his thesis and came back for his final exam. One of his courses had been algebraic topology, and the examiner asked the student for the definition of a covering space; at first, the student was stumped, but on further questioning he came up with an example, the line and the circle. The examiner probed further to ask which covered which, and the student thought the circle covered the line. Disaster! (The solution is to wind the line again and again around the circle, which makes the line cover the circle.)

In the reorganized Ph.D. program, the final Ph.D. oral exam no longer probed all the courses the student had taken. Instead, it stuck to the thesis and related things, and was pretty much a formality, since the student had already shown his or her breadth of knowledge in the new Ph.D. qualifying orals. Under this new setup, the situation described above might not have happened and the student would have passed.

In the Stone department, there was a whole new sequence of required graduate courses. After the master's courses, there were no further required courses for the Ph.D.; the student chose advanced post-Master's courses, took qualifying exams, and completed his or her research. In many cases, faculty prepared mimeographed notes for the new Master's courses: Zygmund prepared notes on analysis; others prepared notes on set theory and metric spaces, as well as several alternative sets of notes on point-set topology (one of which was mine). The latter notes recognized that topology had a central position, comparable to that once held by variable theory. The Stone–Weierstrass theorem made a vital appearance there. Weil complained that the classical definition of determinants should be replaced by a definition making some use of Grassman algebras, which I presented in some lecture notes. The Master's program represented a new, systematic view of mathematics.

This new model of emphasis cropped up all over, and there were new advanced courses. Weil's book on the foundation of algebraic geometry appeared in the Colloquium Series of the AMS. He lectured on the subject; I read and listened, and in my reading, I noted that this version of algebraic geometry could make considerable use of category theory, an observation that I regrettably did not follow up (later, Grothendieck developed the same observation). Weil also conducted a seminar on current literature, inspired by an older Parisian seminar taught by Hadamard. The principle of the seminar was that each student should report on a current paper not in his field of specialty; the student reports would be open to criticism. It is interesting that Weil saved his most devastating comments for those students he knew to be the ablest.

Zygmund taught analysis and encouraged many notable students, such as Alberto Calderón and Elias Stein. He impressed upon all of them the importance of harmonic analysis. Chern taught differential geometry using differential forms, of course. Isadore Singer and other students learned from this approach, which was not yet readily available in texts. It is reported that at one time a student told Weil that he (the student) did not understand these differential forms, to which Weil responded by going to the blackboard and writing down the Greek letter omega (ω); I supposed he intended that this standard symbol for a differential form might recall the idea. Spanier and I alternately taught algebraic topology; in particular, I struggled with the still-mysterious properties of those spectral sequences found by Leray and now used by all topologists.

Mathematical discussions continued in the corridor and at the daily tea. For example, I recall trying to persuade some students at tea of the logical existence of a maximal atlas for any differential manifold — logic was not, as before and afterwards, isolated from mathematics.

In this hothouse atmosphere, ideas and proofs were prominent and many graduate students flourished, including Isadore Singer, Bert Kostant, Richard Kadison, John Thompson, and many others.

As an illustration, I will describe Thompson's work: He had been an outstanding undergraduate in mathematics at Yale, and the reputation of the new department probably attracted him to Chicago for graduate study. Shortly after his arrival, I wanted a chance to do something different, and decided that group theory was due for a revival; hence, I taught a two-quarter course on group theory. By the end of the second quarter, in which I had tended to emphasize infinite groups, I had essentially come to the end of my knowledge on group theory. Thompson, who was in the course, came to me to say that he wished to write a thesis on group theory. I encouraged him, but did not trouble to say that my own knowledge of the subject was somewhat limited. But not to worry — I arranged for eminent theorists, such as Richard Brauer, Reinhold Baer and Marshall Hall, to visit Chicago. Each Saturday morning, I listened to Thompson tell me what he had been up to with groups; the subject fitted his interest, and he chose his own problems.

At the time, André Weil was lecturing in group representations with particular attention to the so-called Chevalley groups. One of Weil's students heard what Thompson was up to, and took him aside to tell him that he should not be studying finite groups alone, but should look at representations as well. However, Thompson persisted, and turned out a Ph.D. thesis that settled an outstanding problem of finite groups, a construction of a normal p-complement for certain finite groups.

Adrian Albert was also interested in finite group theory, a subject active at Chicago from the very beginning in 1892, and found special funding to organize a special year on group theory at Chicago. Thompson and Walter Feit, a recent Ph.D. of Richard Brauer at Michigan, were two of the participants; during the year, they solved a famous problem by proving the Odd Order Theorem: A finite simple group that is not cyclic cannot be of odd order, which was a famous conjecture of Burnside. This result was the starting point of a major effort to classify all finite simple groups.

Group theory, as here exemplified, was just one of the topics that flourished during the Stone Age at Chicago. The people involved — the students and professors — were excited by the pursuit of new ideas in mathematics, which was, in all, a happy combination of talent, circumstance, and leadership.

Saunders has given us an account of his life and work in this autobiography, and through his publications, his students, and his collaborators, he has left a splendid, influential record. Thank you.

References

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