

CELEBRATIO MATHEMATICA

Saunders Mac Lane

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PREFACE TO

“SAUNDERS MAC LANE: A MATHEMATICAL AUTOBIOGRAPHY”

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No man could so stimulate others unless, alongside an incisive intellect, he was possessed of enthusiasm and warmth, a deep interest in his fellow man, and a sympathy the more real for being unsentimental. Those who proudly call themselves his friends know these things: others will infer them in reading [his works].

— Max Kelly¹

Saunders Mac Lane has been my teacher, mentor and model almost from the beginning of my mathematical life. It is a relationship I've cherished. He has been for me a figure of great honesty and integrity, who worked hard to advance research and to serve the mathematical community. His belief in the good, the right and the rational, his care for the essence of mathematical ideas, his powerful enthusiasm and his essential optimism were, and are, deeply attractive to me.

Nearly everything about Saunders-in-action was colorful, starting with the red-and-green plaid sports coat (the Mac Lane tartan, of course) and red pants that he would wear for important occasions. Perhaps a few anecdotes and reflections from my experience of him over 40 years will help the reader appreciate this color.

First encounter

I first met Mac Lane — in a sense I'll make precise — in 1963. He was one of the most important figures in the University of Chicago Mathematics Department, or indeed in American mathematics: His first student, Irving Kaplansky, was Chair of the department, and two other students were on the faculty — one, John Thompson, a Fields Medalist. Mac Lane was an inventor of group cohomology, a founder of homological algebra and category theory, known for the Eilenberg–Mac Lane spaces in topology. He was past President of the Mathematical Association of America, and he would soon be Vice President of the National Academy of Sciences, member of the Board governing the National Science Foundation, and President of the American Mathematical Society as well.

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1: From Saunders Mac Lane: *Selected Papers*, edited by I. Kaplansky, Springer-Verlag 1979.

I knew none of this. I was sixteen, an early entrant to the University, an uneven student with a great enthusiasm for mathematics. It was the beginning of my second quarter, and I was scheduled to start a basic linear algebra class that morning. I happened to arrive a little early, settled down in the first row of the class, and sank peacefully into a daydream. Being so new, I wasn't surprised not to know the other students who settled in around me, and I didn't know the teacher that I'd have. In due course, Mac Lane walked in and began lecturing. His style was lively and colorful, and I was immediately interested — but almost at once aware that I'd made a big mistake: this was not an undergraduate linear algebra course, but an advanced graduate course on category theory. I'd come an hour early.

I understood nothing whatever after a few moments, but was far too embarrassed to get up and leave — instead I sank into daydreams, glassy-eyed. Mac Lane, who prided himself on paying attention to his class, later told me he thought he could always see who was following and who was not. In a moment like a thunderclap, I looked up from my seat and found him pointing directly at me from across the room. "You!" he said peremptorily, "you don't believe this proof, do you?" Belief and disbelief were equally beyond me; I sat petrified. He advanced toward me, and I don't know what I imagined — that he would pick me up by the scruff of my neck and throw me from the room? He stopped, turned back to the board, and proceeded to explain the proof to satisfy me. Of course, I still understood nothing — but I sat in rapt attention.

Fortunately the class ended soon, and as students asking questions surrounded him, it was easy for me to slip out. I didn't tell Saunders this story until many years afterwards, when I had the privilege of re-enacting it (from the other side) in a lecture at the conference in honor of his seventieth birthday. Needless to say, the event hadn't left a trace in his memory, though it remains sharp for me to this day.

Saunders and tolerance

Saunders believes strongly in principles, in the rightness of right positions. I never once saw him personally intolerant, but he could sometimes be direct and candid to the point of offending. People whose judgment I respect have felt injured by what he said, and sometimes by the bluntness of his expression. In some way perhaps he didn't appreciate the magnitude of his position in mathematics, or the seriousness with which people took him. In a lesser personage some of his extreme positions might have been regarded as charming eccentricities. But given Saunders' stature, they could injure, and he might have been more cautious.

An event from late in Saunders' life may give a bit of the flavor. It was a special session run by him and Richard Askey at the Joint Mathematics Meeting in 1999, a session boldly entitled "Mathematics education and mistaken philosophies of mathematics." The audience was enormous. I found the title charming (and still find it so, even now as I become more involved with ideas in K-12 education), and I imagine that Saunders meant it to be controversial but playful. Predictably, it annoyed and needled some practitioners. Saunders began the session with introductory remarks that I found fascinating: he said that he now considered the extent of his own emphasis on category theory as a tool for learning and teaching mathematics to have been too extreme. This humbleness may have helped soften the critical tone of the session.

Saunders and Sammy

One of Saunders’ great mathematical friendships and collaborations was with Samuel Eilenberg (widely known as Sammy, or even S^2P^2 : “Smart Sammy, the Polish Prodigy”). I got to see them in action together only once, at the AMS Summer Research Institute on Category Theory at Bowdoin College, in 1969. They had special status at this three-week conference, not only as the senior members, but also as the very founders of the subject. So, when they began discussing its origins one evening after dinner, everyone gathered around to listen.

I dearly wish I could recall the substance of their debate, but I don’t; only my sense of the contrast in the two men’s styles stays with me. Sammy drew Saunders out and egged him on, always slightly evasive and mocking; Saunders, whose father and grandfather were Congregational ministers, seemed to feel that, since his view was right, his view would prevail. Once he had stated it, all he could do was bang his fist. The devious and sophisticated European versus the innocent but honest American? That’s how it seemed to me at the time (maybe I was a little innocent myself). A loyal student, I was rooting from the beginning for Saunders’ point of view, but I came away feeling that he was trounced in the contest.

Being Saunders’ student

After flirting a while with operator theory (Paul Halmos and Felix Browder were my teachers) and group theory (learned from Jon Alperin and Otto Kegel), it was finally time for me, by now a second-year graduate student, to settle on an area for a PhD thesis. I obsessed about how to make the choice. A close mathematical friend, Joe Neisendorfer, explained to me an algorithm: forget the topic, look around the faculty for the person you like the most. It didn’t take me long to choose Saunders.

I wouldn’t say I ever felt personal intimacy with Saunders, but he did go out of his way to make me and other students feel welcome in more than his office. Saunders and his late wife, Dorothy, had a small but comfortable cottage in the Indiana Dunes, a beautiful area on the shore of Lake Michigan about an hour south of Chicago, and they occasionally invited students to spend an afternoon there. Saunders was an enthusiastic sailor, and I can report, from a ride in a small sailboat on rough water, that he was ready to provide needed instruction not only in mathematics, but also on how to handle the absence of a toilet — or any privacy — in that difficult situation.

If you look at the list of Saunders’ 39 students, you’ll see that Irving Kaplansky, who worked on valuation theory of fields, came first; I’m near the end, with a thesis on noncommutative rings. Along the way are such people as John Thompson (finite groups), Anil Nerode (logic and computation), and Robert Szczarba (algebraic topology). How did this variety come about?

Perhaps the answer lies in Saunders’ hospitality to these many ideas. He wanted to learn finite groups, and taught a course on them. By the end of the course he’d decided that he’d never really understand the subject, but in Thompson he found a fabulously strong student. Saunders might have tried to turn such a student toward interests close to his own, but I think he would not, on principle: he was happy to encourage his students to do what excited them.

Saunders has followed an interesting, curving trajectory through mathematics, from logic and foundations to field theory and the beginnings of homological algebra, through topology to category theory, with

smaller diversions along the way into Hamiltonian mechanics, finite groups, and many other subjects. Perhaps his students, or many of them, could be described as coming off on the tangents to this path, a kind of developable surface reaching broadly across mathematics. Altogether, Saunders has more than 1,000 mathematical descendants listed on the [Mathematics Genealogy Project](#).

Some other aspects of Saunders are also reflected in his students: Saunders was always active on behalf of the community, whether as Chair working to build the department at the University of Chicago or, near the end of his career, as member of the National Science Board or as manager of the elaborate system of reports for the National Academy of Sciences. Many of his students and grand-students have followed him into this willingness for public service. When I was worrying about whether to move to my current position at MSRI, he was one of the first people I called on for advice and blessing, and he gave both.

Returning to the more fundamental matter of being Saunders' mathematical student: I tried for a while, dutifully, to find a thesis topic in category theory, Saunders' passion in that part of his life. But I failed; somehow, the things I read and learned in that domain just didn't inspire me. When I developed an interest instead in a problem on non-commutative rings posed by a visitor of Herstein, the young Chris Robson, Saunders could easily have washed his hands of the project. He did not: though it was far from his current area of interest, he welcomed what I had done, and painstakingly read draft after draft of my thesis.

Saunders' mode of instruction in thesis-writing bears mention. I had written a couple of papers, jointly with Robson, of which my thesis results were partially an extract. Robson cared a lot about exposition, and so (learning from Saunders among others) did I. We'd gone through many drafts, and I thought the writing pretty polished. Saunders did not. He began at the beginning and worked his way through the thesis until he'd compiled a list of exactly 25 substantive suggestions. Then he stopped, and returned the document to me for an overhaul. When I had finished making the corrections he'd flagged and all their analogues, I gave it back to him, eager to be done. But... after a week or so I got a second list of exactly 25 more suggestions. The third list was a bit shorter, and Saunders allowed the process to converge before I got too frustrated.

It must be clear by now: over these forty years I've learned many lessons from Saunders. I'm deeply grateful to him.

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