CELEBRATIO MATHEMATICA

Kai Lai Chung

RONALD GETOOR

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Kai Lai Chung was an original, in the best sense of the meaning of that expression. He had a great enthusiasm for mathematics and life that was contagious. Chung was one of the leading probabilists of the second half of the twentieth century. He had strong opinions about mathematics and other subjects, which he expressed vigorously and emphatically. He could enliven any discussion or meeting with his lively personality, enthusiasm and exuberance.

He appreciated going to the best restaurants, having fine wine and staying at the best hotels. In 1970 we were both speakers at the International Congress of Mathematicians in Nice, and he took great delight in kidding me that he was staying in a five-star hotel while I was only in a four-star hotel. Actually, both hotels were quite old and very ornate, and they clearly had seen better days. They were on the famous Promenade des Anglais, which was a busy fume-filled noisy motorway at the time. The Mediterranean was just across the street from the hotels, the beach consisted of stones, and was not particularly inviting. So one afternoon we decided to make an excursion by hydroplane to Cannes in order to see the famous beaches and hotels at this well-known resort area. It almost lived up to its reputation. Anyone meeting Chung was not likely to soon forget him.

He made fundamental contributions to many areas of probability theory, ranging from sums of independent random variables to probabilistic potential theory. He was rigorous and demanding in his approach to mathematics. He had little patience with arguments that were incomplete. He could become particularly annoyed with authors that slurred over crucial technical details. His bibliography lists more than 125 papers and eleven books. Moreover, he was an excellent expositor. For example, his book *Lectures from Markov Processes to Brownian Motion* [1982] is the best introduction to the circle of ideas associated with the names of Hunt, Meyer and Dynkin. After it appeared, it was the book I recommended for my students to begin reading. The little book *Green, Brown and Probability* [1995] is a delight. He took great pleasure and pride in his use of language. The prose in his books and papers is often elegant and engaging. I particularly enjoy his use of asides, especially in his books.

I first met Kai Lai in the spring of 1955. It was during my first year as a Fine Instructor at Princeton, and he gave a talk in Feller's probability seminar. I believe that he was talking about Lévy's work on chains all of whose states were instantaneous. This was quite new and somewhat disturbing at the time. Evidently he had discussed Lévy's results with Doob, and what I remember is how he excitedly exclaimed, "Doob kept asking, 'Where is the random variable?'" I must confess that I did not really understand the issue at the time. In any case, I was impressed by his enthusiasm and excitement. He certainly was very different from any of the Chinese that I had known previously. In preparing these remarks, I realized that during

my two years at Princeton I don't remember any other speaker in Feller's seminar, although there must have been one most weeks during the academic year. Yet I remember Kai Lai in the seminar very vividly after nearly 55 years. This speaks to how impressed I must have been by Kai Lai.

In those years 1955–56, Feller was working on his boundary theory, and I was trying to understand it in the hopes of finding a topic for research. I don't remember if I mentioned this to Kai Lai, but a little later he wrote me several times asking question about Feller's boundary-theory papers. I answered to the best of my ability. I had managed to work through the details of Feller's papers, but I never developed a real feeling for the underlying issues.

I don't recall having much more contact with Kai Lai during the next few years, although I must have met him and spoken with him at meetings. Then, I spent the academic year 1964–65 visiting Stanford, and this is when I got to know Kai Lai. Actually, it was Sam Karlin who invited me to spend the year at Stanford, and made all the arrangements. Kai Lai had come to Stanford in 1961, and as it turned out I spent a good deal more time talking with Kai Lai than with Sam. Kai Lai had become interested in potential theory and its relationship with Markov processes. By then I had become interested in Markov processes in the spirit of Hunt, Dynkin and Meyer. I lectured on this subject for two quarters, covering some of the topics that would appear eventually in my 1968 book written jointly with Bob Blumenthal. Kai Lai sent several of his students to the lectures. I remember Naresh Jain and Art Pittenger. I also discovered a few years ago that Tom Kurtz had attended, but I must confess that I don't remember him being in the class. During this year at Stanford, I got to know Kai Lai, and we became good friends. We often had lunch together, usually at a place called the Barn located in the Stanford shopping center. It was what would be called a food court these days, with various ethnic-food places around the periphery, with tables in the center. It was a great place where one could linger and discuss mathematics, with an ample supply of paper napkins to scribble on. Occasionally Sam Karlin would join us. At the time there was a joint probability/statistics seminar between Stanford and Berkeley that met once a month, alternating between the two campuses. When it was at Berkeley and the topic was of interest to us, we would travel to Berkeley together. My recollection is that I usually drove.

Kai Lai had invited Marcel Brelot to visit Stanford during the spring quarter of 1965 and lecture on classical potential theory. Both Kai Lai and I attended Brelot's lectures which, to the best of my recollection, followed more or less his well-known *Éléments de la Théorie Classique du Potentiel* [1959], which appeared in "Les cours de Sorbonne" lecture notes. During the first several lectures the class room was completely filled, but by the end of the quarter Kai Lai and I were the only attendees. It was a great opportunity for me not only to get to know Kai Lai, but also to become friends with Brelot.

In 1966 I moved to San Diego, and after that Kai Lai and I exchanged numerous visits. During his visits to San Diego, Kai Lai took great pleasure in visiting the San Diego Zoo. He was very interested in animals, and usually tried to work-in a visit to the Zoo when he was in San Diego. He enjoyed having lunch on the patio of the La Valencia Hotel in downtown La Jolla, and we would try to have lunch there whenever he was in town. He would usually have one and only one beer with his lunch.

In the seventies and eighties, we had an extended correspondence on various topics concerning Markov processes and related subjects. This correspondence flourished especially during the period he was writing his book, *Lectures from Markov Processes to Brownian Motion* [1982]. Kai Lai's letters often

ended with the admonition "Answer at once," or just "Answer," or words to this effect. I particularly enjoyed this exchange of ideas, and I learned much in my attempts to answer some of his questions. I am happy that he and I were able to collaborate on a short paper, giving the probabilistic meaning of the condenser charge in classical potential theory, and that he chose to include this paper in his volume of selected works. Much later, after he retired, Kai Lai told me that he had decided to destroy all of his correspondence, and after I retired I did the same.

In the late nineties, my daughter was a graduate student in Computer Science at Stanford. Kai Lai had retired by then, but when I visited her I would call him, and often he would invite me to his house in the afternoon for a beer or tea, and conversation. It was during one of these visits that he told me about destroying his correspondence. I was honored in 2000 that he came to San Diego and spoke at my retirement. During that visit we had the opportunity to have lunch on the patio of the La Valencia once again, and as before he had one beer with lunch. This was the last time we met.

In reading his biography in the *IMS Bulletin*, I learned that he had wide-ranging and intimate knowledge of opera. I am also an opera fan and interested in opera, but this never came up in our discussions. In fact the main topics I remember discussing with him were mathematics, the politics of mathematics and other mathematicians.

Finally let me say a few words about his mathematics. Here I shall make some general remarks on his work, since I already have made more technical comments on several of his papers in the volume of his selected works. My comments only apply to that portion of his oeuvre with which I am most familiar, roughly corresponding to the period from the late sixties to the early nineties. Although it is obviously an oversimplification, I think that there are two closely related ideas or, perhaps better, themes underlying some of his most important work during this period. Namely, the ideas of reversing the direction of time in a Markov process, and of using last exit times.

Here and in the remainder of this discussion, the term Markov process means more precisely a continuous-parameter temporally homogeneous Markov process on a reasonably general state space, for example, a locally compact space with a countable base or some generalizations of such spaces. The parameter set is usually the nonnegative reals, and in addition the process is assumed to be strong Markov with right-continuous paths having left limits.

Since the definition of the (simple) Markov property is symmetric with respect to past and present, this property is preserved if the direction of time is reversed. But in general, the temporal homogeneity is not, nor is the strong Markov property, and clearly right continuity with left limits becomes left continuity with right limits. One of the reasons for the interest in reversing a process is that, in Hunt's work on duality theory for Markov processes, he proved the existence of a temporally homogeneous right-continuous strong Markov dual process under hypotheses that were somewhat opaque and difficult to verify. Later, in [Blumenthal and Getoor 1968] for example, the existence of such a dual process was taken as a hypothesis in developing duality theory. It was clear that in some sense the dual process was just the original process with time reversed. Thus, in addition to the intrinsic interest of the possibility of reversing time, it was important to try and understand the relationship between the reversed process and the dual process.

In order to preserve the temporal homogeneity of the reversed process, it was necessary to reverse time from a random time ζ, so that the reversed process took the form $\hat{X}(t) = X(\zeta - t)$ for $0 < t < \zeta$, where X is the original process and ζ is a random time, subject to certain hypotheses that guaranteed that $\hat{X}$ was temporally homogeneous. Initially ζ was taken to be the "lifetime" of X, but later this was extended to more general times. Hunt considered this problem in the case of discrete-parameter Markov chains in 1960, and Chung considered the case of a class of continuous-parameter Markov chains in 1962. In the mid-sixties, this work was extended to the class of Markov processes described above, in a series of papers associated with the names of Ikeda, Kunita, Nagasawa, Sato, S. Watanabe and T. Watanabe. However, these last-named authors assumed the existence of a pair of semigroups (or resolvents) in duality, the dual semigroup being the semigroup of the putative dual or reversed process. Moreover, in order to force the dual or reversed process into the class of right-continuous strong Markov processes, they replaced $\hat{X}(t)$ by its right limit; that is, the dual process was taken to be $\tilde{X}(t) = \hat{X}(t+)$, so that the path $t \mapsto \tilde{X}(t)$ is right continuous.

A fundamental breakthrough was made in the 1969 paper of Chung and Walsh, *To reverse a Markov process* [1969]. The authors did not try to fit the reversal into the right-continuous strong Markov framework as in most previous work. They worked directly with the left-continuous process $\hat{X}(t) = X(\zeta - t)$, as defined above. Much more importantly, they did not assume the existence of a dual semigroup; rather, part of their construction of a good dual process was constructing a good dual semigroup. Moreover, they realized that the strong Markov property had to be modified when the process had left-continuous paths. They defined a new property that they called the moderately strong Markov property, and showed that this new property was the proper replacement of the strong Markov property when the paths were left continuous. P. A. Meyer wrote that this paper of Chung and Walsh "made very important (*très grand*) progress in the theory of reversing a Markov process." Paraphrasing slightly, he went on to say that the originality of this paper consisted precisely in using the natural left-continuous reversal, and proving that it satisfied a "moderate" Markov property that is the natural form of the strong Markov property for left-continuous Markov processes. Of course, this paper contains much more than what I have mentioned here, and I have only made superficial remarks about those parts I have discussed. In particular, the construction of a good dual semigroup was technically difficult, and introduced methods that had not been used previously in the work on Markov processes; for example, the use of essential limits.

The somewhat awkward name "moderately strong Markov property" quickly evolved into the more succinct "moderate Markov property." This is not a weak form of the strong Markov property, but a distinct property that is appropriate for left-continuous processes. (More logical terminology might be "right Markov" and "left Markov" for "strong Markov" and "moderate Markov", respectively.) The relationship between these properties was clarified by Chung in his 1972 paper, *On the fundamental hypotheses of Hunt processes* [1972]. Left-continuous moderate Markov processes were studied as processes in their own right in a joint paper by Chung and Glover in 1979, *Left-continuous moderate Markov processes* [1979]. Since the mid-eighties, the existence of a left-continuous moderate Markov dual process has been used extensively in the study of the potential theory of Markov processes. There is an updated and much expanded exposition of the Chung–Walsh theory in the second half of their 2005 book *Markov Processes, Brownian Motion and Time Symmetry* [2005].

Regarding the use of last exit times, Chung had already made good use of such times in his work on Markov chains. In the work of Hunt, and for the next ten years following Hunt, a key concept was that of an optional time, of which the fundamental example is a first entry (or hitting) time. Last exit times were hardly considered, if at all, during this period. But it is clear that a last exit time for X is just a first entry time for the reversed process $\hat{X}$ defined earlier. Thus, oversimplifying again, the study of last exit times may be reduced to that of the more familiar first entry times, but for the reversed process.

In his paper *Probabilistic approach in potential theory to the equilibrium problem* [1973], Chung made spectacular use of last exit times to obtain a beautiful formula for the equilibrium distribution of a set, in terms of the last exit distribution from the set and the potential kernel of the underlying process. Since I have commented on this paper in some detail in the volume of his selected works, I will confine myself here to this: This paper was very original in its method, and this was immediately recognized by workers in probabilistic potential theory.

Of course, the above is just a small sample of the many important contributions that Kai Lai Chung made to the theory of Markov processes and probabilistic potential theory.

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