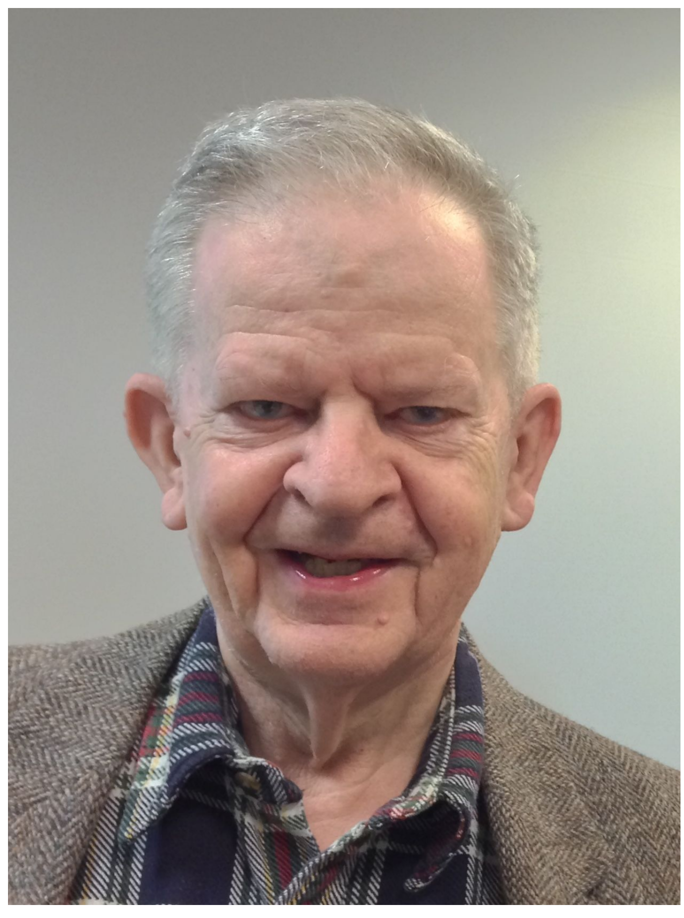
Liber Amicorum
Richard “Dick” Allen Askey
— a Friendship Book —

from Dick’s colleagues and friends
September 15, 2019
Madison, Wisconsin

Dick Askey

Liber Amicorum
Dick Askey
Second Edition

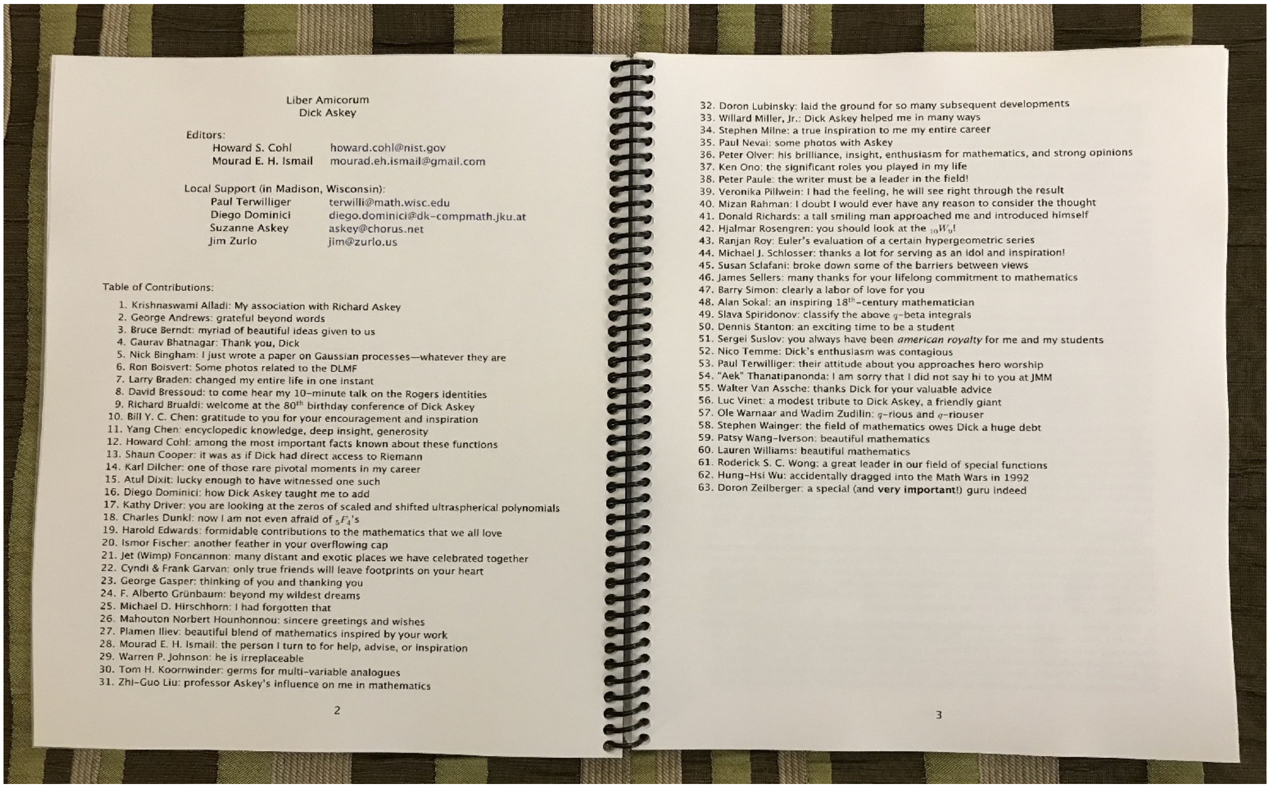
Editors:
Howard S. Cohl howard.cohl@nist.gov
Mourad E. H. Ismail mourad.eh.ismail@gmail.com

Local Support (in Madison, Wisconsin):
Paul Terwilliger terwilli@math.wisc.edu
Diego Dominici diego.dominici@dk-compmath.jku.at
Suzanne Askey askey@chorus.net
Jim Zurlo jim@zurlo.us

Table of Contents:

Foreword to Second Edition

The original idea for giving a friendship book to Dick Askey during the Fall Sectional AMS Meeting in Madison, Wisconsin on September 14–15, 2019, occurred on September 4, 2019 in an email discussion between Mourad Ismail and Howard Cohl. After that, Howard and Mourad sent a bulk email to Dick's colleagues, informing them of Dick's grave situation. Henceforth, there was a great outpouring of warmth and gratitude for Dick, and many contributions were received. Over the course of 10 days, these contributions were carefully assembled into a PDF document.



Liber Amicorum
Dick Askey

Editors:
Howard S. Cohl howard.cohl@nist.gov
Mourad E. H. Ismail mourad.eh.ismail@gmail.com

Local Support (in Madison, Wisconsin):
Paul Terwilliger terwilli@math.wisc.edu
Diego Dominici diego.dominici@dk-compmath.jku.at
Suzanne Askey askey@chorus.net
Jim Zurlo jim@zurlo.us

Table of Contributions:

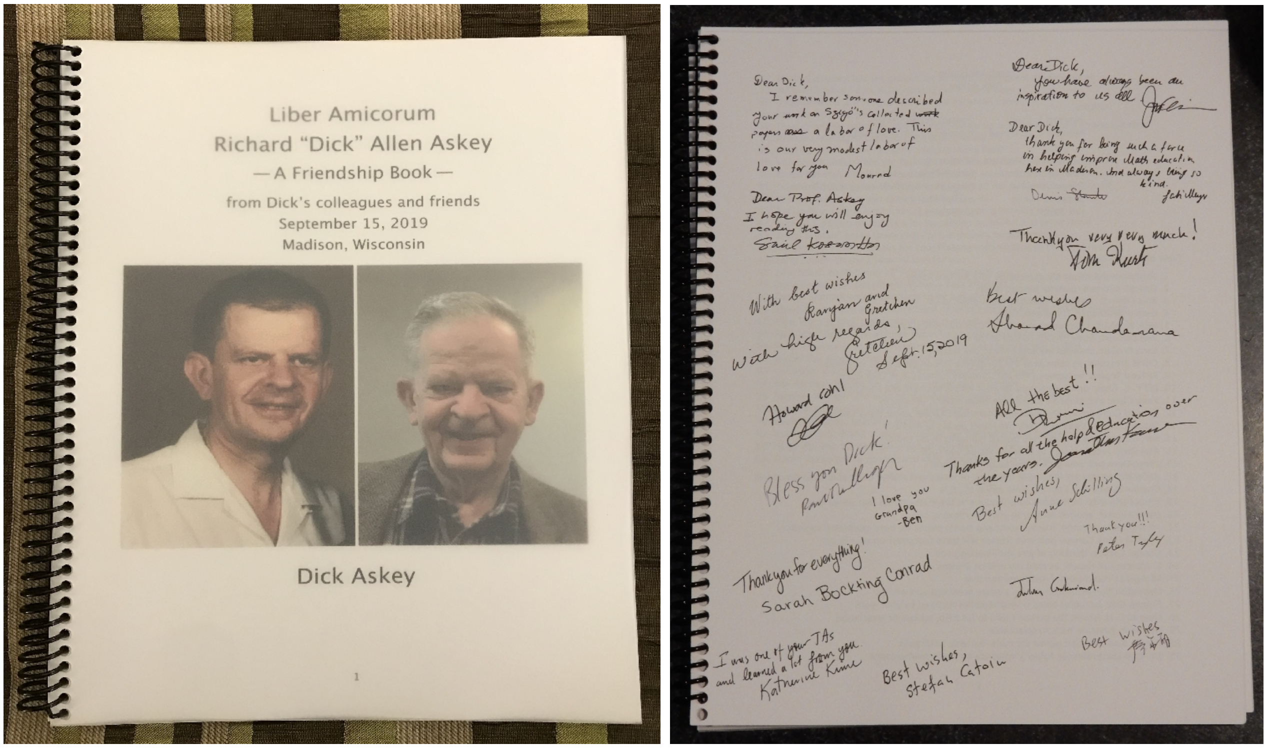
2

3

Photo of original table of contributions for the Liber Amicorum, a Friendship Book for Dick Askey. Photo taken by Howard Cohl.

After attending the last afternoon talk in the *Special Session on Special Functions and Orthogonal Polynomials* on Saturday, September 14 (organized by Sarah Post, University of Hawaiʻi at Mānoa, and Paul Terwilliger, University of Wisconsin–Madison), Dick Askey's son, Jim Zurlo, drove Mourad Ismail and Howard Cohl to a FedEX store in Madison, Wisconsin. It was then arranged to print what ended up being a beautifully crafted First edition of the Liber Amicorum, a Friendship Book for Dick Askey. It contained 63 contributions, with 62 color photographs and was 106 pages long.

On Sunday, September 15, after the last talk of the morning session, two groups of mathematicians boarded two separate cars to drive to a chapel at Oakwood Village Prairie Ridge, about 10 miles northeast of downtown Madison. The first car was driven by Thomas Kurtz, Emeritus Professor at the University of Wisconsin. He drove Mourad Ismail and Dennis Stanton to the chapel. The second car was driven by Peter Tingley, Loyola University Chicago, who drove Howard Cohl, Diego Dominici, and Anne Schilling to the chapel. Ranjan Roy and his wife drove to the chapel in a separate car from Beloit, Wisconsin.



A photo of the cover (taken by Howard Cohl) and a photo of the signature page (taken by Jim Zurlo, Dick's son) of the original Liber Amicorum, a Friendship Book for Dick Askey. This original printed book was presented to Dick in Madison, Wisconsin on the morning of Sunday September 15, 2019. On that same morning, the Friendship Book was signed by the following individuals. First column: Mourad Ismail, unknown signature, Ranjan and Gretchen Roy, Howard Cohl, Paul Terwilliger, Ben Hinkel (Suzanne's son and Dick's grandson), Sarah Bockting-Conrad, Katherine Kime, Stefan Catoiu. Second column: Jin-Yi Cai, unknown signature, Dennis Stanton, Tom Kurtz, Sharad Chandarana, Diego Dominci, Jonathan Kane, Anne Schilling, Peter Tingley, Julien Gaboriaud, Hanmeng (Harmony) Zhan. Cover page photo taken by Howard Cohl; Signature page photo taken by Jim Zurlo, Dick's son.

After arriving at the Chapel, Dick had not arrived yet, but his family was there as well as several colleagues from the University and elsewhere. Most of the people who were present (there were about 30 people) signed an empty page in the front of the Friendship Book. Dick was sleeping, but was about to wake up and before long he arrived in a wheel chair with a nurse. He was positioned at the front of a circle of chairs with his wife Liz next to him, as well as his son Jim and daughter Suzanne.

The whole emotional ceremony lasted approximately 45 minutes. When Dick was ready to start, Suzanne made some short comments. Suzanne pretty much ran the whole show. Howard stood up, gave a short description of the contents of the Friendship Book, and presented it to him. Dick, who wasn't able to move quickly, leafed through the pages and examined the front cover and the table of contents. Suzanne leafed through the book for Dick and showed him some of the many historical photos of Dick and his colleagues and friends, which were contained within the book.

Howard then read out loud the full list of people who had provided written contributions to the Friendship Book. (After Howard's listing of the contributors, Mourad pointed out

l to r: Mourad Ismail, Jim Zurlo (Dick's son), Liz Askey (Dick's wife), Dick Askey, and Suzanne Askey (Dick's daughter). Photo taken by Howard Cohl.

Dick Askey and Suzanne Askey browsing the Liber Amicorum.
Photo taken by Howard Cohl.

that Howard had left out mentioning his own contribution. Howard located and then affirmed its existence, which brought out laughter in the room.) Dick seemed to be quite emotional during these moments. Afterwards, Mourad gave a short speech about the idea of the Friendship book, describing all the people who responded to our request for contributions, and that they all love Dick, and especially Mourad himself who thanked Dick for all the difference Dick had made in so many people's lives. The whole while Suzanne was visibly comforting Dick.

Mourad then introduced Diego Dominici who presented the OPSFA Lifetime Achievement Award to Dick. (This had previously been shown at the OPSFA-15 meeting in Hagenberg, Austria.) Several people spoke and made some statements about Dick and to Dick, and

Dick Askey's OPSFA Lifetime Achievement Award, presented to Dick Askey by Diego Dominici in Madison, Wisconsin on Sunday September 15, 2019. Photo taken by Peter Paule.

asked questions of Dick. Dick said a few words about the OPSFA meetings. Both Mourad and Dick talked about the history of the OPSFA meetings and in particular, OPSFA-1, Bar-le-Duc, France, 1984. Dick mentioned that there was fireworks during that meeting! Dick also referred to his Bronze bust of Srinivasa Ramanujan, one of ten copies cast in 1983, which has been recently donated by Dick's family to the Department of Mathematics at the University of Wisconsin—and will be installed in their 9$^{\text{th}}$ floor lounge.

Dick, Suzanne Askey and Jim Zurlo made several extended comments. Suzanne asked Howard to read some contributions from the Friendship book and Howard read the selected contributions of George Andrews, Barry Simon, and Doron Zeilberger. Dick also reminisced about the fact that he got the idea for the Askey-scheme at an Oberwolfach meeting in 1977 on "*Combinatorics and Special Functions*". He mentioned that Michael Hoare, in connection with his lecture, distributed copies of a sheet which contained in graphical way, a part of the present Askey scheme, and which was received very enthusiastically by the audience.

Note that a second edition of the Liber Amicorum is currently being finalized which will contain at least 85 contributions about Dick and 99 color photographs of Dick.

This final updated Friendship Book now includes some extra contributions. It is a collection of 81 heartfelt anecdotes in 133 pages marking Dick's long service to our community. It is given from those of us who wanted to express our appreciation and gratitude to the friendly, generous, encouraging, enthusiastic, inspiring, insightful, influential, irreplaceable, giant, leader, mathematician and person, Dick Askey.

Note added in Proof (by Suzanne Askey).—Dick lived another 3 weeks and up until the last couple days, every day I read to him from the Liber Amicorum. When I was reading to him he perked up, opened his eyes and focused more than he was able to most of the rest of the time. He was very interested and glad to be hearing all that had been written by mathematicians around the world. He was able to hear all of the contributions before he died. I can't put into words how grateful I am that his last weeks were full of the appreciation, respect, and love from all those who contributed to the Liber Amicorum, and the work put into making it by Howard and the upfront work by Mourad in starting the emails of appreciation for Dick pouring into my email. With much gratitude, Suzanne Askey (Dick's daughter).

From: Krishna Alladi (alladik@ufl.edu)
Contributor: Krishnaswami Alladi: my association with Richard Askey

My Association with Richard Askey,
World authority on special functions and Ramanujan's work

Krishnaswami Alladi

Richard Askey is a world leader in the field of special functions. In an illustrious career spanning six decades, he has greatly influenced the development of that subject by means of his own fundamental research and the work of numerous mathematicians he has groomed. He has also been instrumental, along with George Andrews and Bruce Berndt, in making the mathematical world aware of the wide ranging and deep contributions of Ramanujan. Although I have not collaborated with Professor Askey, I have had close interaction with him for the past four decades. Indeed, my family and I have had the pleasure and privilege of hosting him and his wife both in Madras, India, and in Gainesville, Florida, several times.

I first met him at the Summer Meeting of the AMS at the University of Michigan, Ann Arbor, in 1980. He was giving an hour lecture on the Selberg integral which I attended. I was charmed by his conversational, yet engaging, lecturing style. A vast panorama of the area of special functions unfolded in his lecture, revealing his encyclopedic knowledge of the subject. Of course, he made connections with Ramanujan's startling discoveries, and exhorted everyone in the audience to study the work of the Indian genius. I was working in analytic number theory at that time, but before the end of that decade, owing to the lectures of Andrews, Askey and Berndt that I heard at the Ramanujan Centennial in India in 1987, I entered the world of q, and as Askey would say, I was smitten by the q-disease!

The Ramanujan Centennial was an occasion when mathematicians around the world gathered in India to pay homage to the Indian genius, and take stock of the influence his work has had and the impact it might have in the future. Askey was one of the stars of the centennial celebrations. I organized a one day program during a conference at Anna University, Madras, in December 1987, and he graciously accepted our invitation to inaugurate that conference. He delivered a magnificent lecture on "Beta integrals before and after Ramanujan" in my session. We were also honored to have him give a public lecture at our family home in Madras under the auspices of the Alladi Foundation that my father, the late Prof. Alladi Ramakrishnan, had created in memory of my grandfather Sir Alladi Krishnaswami Iyer.

With my research being focused in the theory of partitions and q series from 1990, we have had a series of conferences at the University of Florida emphasizing this area. Professor Askey has visited Gainesville several times both as a lead speaker at these conferences, as well as for History Lectures and talks on mathematics education during the regular academic year. I have enjoyed every one of his lectures in Gainesville and elsewhere at major meetings. I want to share with you one interesting episode.

In 1995, there was a two week meeting on special functions, q-series, and related topics, at the Fields Institute in Toronto. The first week was an instructional workshop, and the second week was a research conference. I attended the second week. The great I. M. Gel'fand was scheduled to be the Opening Speaker for the research conference. I was looking forward to Gel'fand's lecture since I had heard so much about the Gel'fand Seminar he had conducted in Moscow, and how he would dominate the seminar and cut people down to size. It turned out that Gel'fand could not come to Toronto due to ill health (he was 82 years old). So Askey got up and said that he was the one who had invited Gel'fand, and if the person you had invited is unable to come, then you should give a talk in his place. So Askey gave a masterly lecture on special functions in his imitable style that I thoroughly enjoyed.

I also want to share an episode regarding Askey in the audience for one of my lectures. This was in June 1993 and I was speaking in the Paris Number Theory Seminar at the Institute Henri Poincare on the theme "The combinatorics of words with applications to partitions". Just as I started my lecture, Askey walked in. He keenly followed the lecture, and at the end when my host Michel Waldschmidt asked if anyone had comments or questions, Askey's hand went up. He said: "You gave a very nice talk, but having said that, let me tear you apart." He then proceeded to point out where I could have been more accurate with regard to historical statements as well as statements regarding the depth of various q-series identities. I had started research in the theory of partitions and q-series only in 1989, and so this was among my early talks in the field. And yes, he was right in all the comments he made.

His insight and critical comments have been immensely useful to me in various ways. Starting from the Ramanujan Centennial, I wrote articles annually for Ramanujan's birthday for *The Hindu*, India's National Newspaper, comparing Ramanujan's work with that of various mathematical luminaries in history. I benefited from Askey's comments and (constructive) criticism in preparing these articles. A collection of these articles appeared in a book that I published with Springer in 2013 for Ramanujan's 125$^{\text{th}}$ birthday [4].

By the time the Ramanujan 125 celebrations came around in 2012, I was firmly entrenched in the Ramanujan World, and so was involved with the celebrations in various ways. In particular, owing to my strong association with SASTRA University, I organized a conference at their campus in Kumbakonam, Ramanujan's hometown. We felt that Askey, Andrews, and Berndt, had to be recognized in a special way in Ramanujan's hometown for all they had done to help us understand the plethora of identities Ramanujan had discovered. So *The Trinity* (Askey, Andrews, and Berndt) were awarded Honorary Doctorates by SASTRA University in a colorful ceremony at the start of which they entered the auditorium with traditional South Indian Carnatic music being played on the *Nadaswaram*, a powerful wind instrument. Askey enjoyed the ceremony but felt that the music was too loud; but that is how the Nadaswaram is, since it is played in festivals attended by a thousand people or more!

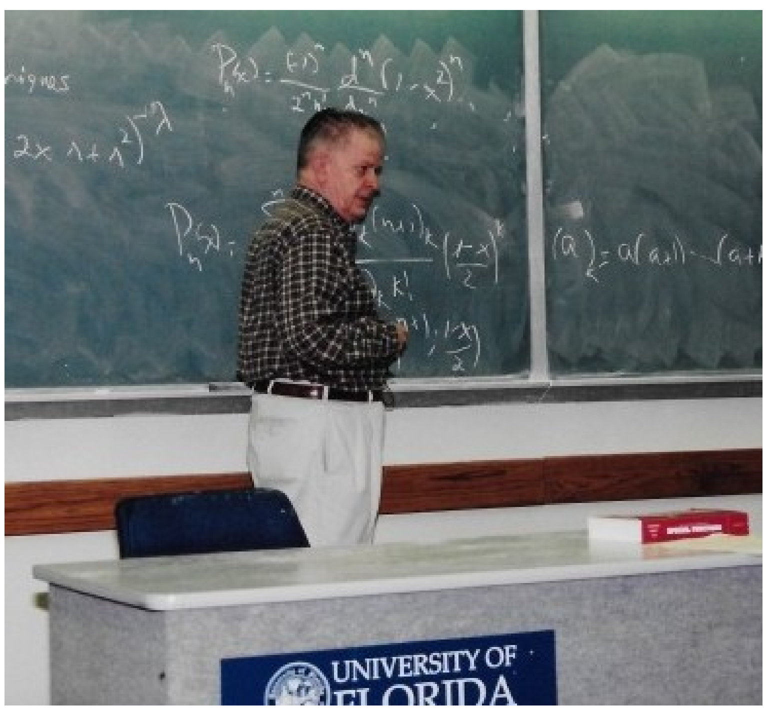
There is much that can be said of Dick Askey. But I will conclude by emphasizing, that in spite of his eminence, he is a very friendly and helpful person. It is rare to find eminence combined with humanity, and Askey has this precious combination which has been beneficial to so many of us.

Krishna Alladi
Department of Mathematics
University of Florida
Gainesville, FL 32611 USA
email: alladik@ufl.edu
(Article written in Sept 2019)

Richard Askey giving a talk on Ramanujan at the Alladi's family home in Madras, India, on December 19, 1987, during the Ramanujan Centennial. The photograph behind Askey is that of Krishna's grandfather Sir Alladi Krishnaswami Iyer.

Askey responding to a question after his talk on Ramanujan at the Alladi family home in Madras, India, in December 1987. Next to Dick is Krishna's father, the late Prof. Alladi Ramakrishnan.

Askey and his wife with some delegates for the Ramanujan Centennial at the Alladi family home in Madras, India, in December 1987. From L to R: David Bressoud, Bruce Berndt, Mrs. Askey, Alladi Ramakrishnan, George Andrews, Krishna Alladi, Dick Askey, Basil Gordon, and M. V. Subba Rao.



Dick Askey giving a featured History Lecture at the University of Florida, Gainesville, Florida, on March 21, 2005.

Dick Askey in conversation with Mathura Alladi (Krishna's wife), Lalitha Ramakrishnan (Krishna's mother), and Alladi Ramakrishnan (Krishna's father), during a party in honor of Dick at the Alladi home in Gainesville, Florida, March 21, 2005.

Dick Askey at the Alladi home in Gainesville, Florida, March 21, 2005. From L to R: Krishna Alladi, John Thompson (group theorist, Fields medalist), George Andrews, Dick Askey, and Alladi Ramakrishnan (Krishna's father).

Richard Askey receiving an Honorary Doctorate from SASTRA University in Kumbakonam (Ramanujan's hometown), during the Ramanujan 125 celebrations, December 15, 2012.

Richard Askey speaking after receiving an Honorary Doctorate from SASTRA University in Kumbakonam (Ramanujan's hometown), during the Ramanujan 125 celebrations, December 15, 2012.

From: Horst Alzer (H.Alzer@gmx.de) and Man Kam Kwong (mankwong@connect.polyu.hk)
Contributor: Horst Alzer & Man Kam Kwong: influenced our research tremendously

Unfortunately, we never met Richard Askey personally, but his books and papers have influenced our research tremendously. We have written several papers on inequalities for trigonometric sums and we have quoted his work many times. His contributions to this subject were (and still are) very helpful and inspiring for us.

One of Askey's favourite results was the following theorem which was published by Vietoris [128] in 1958.

If

$$a_0 \geq a_1 \geq \cdots \geq a_n > 0 \quad \textit{and} \quad 2ka_{2k} \leq (2k-1)a_{2k-1} \quad (k = 1, ..., [n/2]),$$

then

$$\sum_{k=1}^{n} a_k \sin(kx) > 0 \quad \textit{and} \quad \sum_{k=0}^{n} a_k \cos(kx) > 0 \quad (0 < x < \pi).$$

Askey's attempts to call attention to this result are described in detail in [23].

In 1974, Askey and Steinig [37] offered a simplified proof of Vietoris' inequalities and showed that this result can be applied to estimate the zeros of certain trigonometric polynomials and to obtain new positive sums of ultraspherical polynomials. Moreover, they presented the following interesting counterpart.

If $(2k-1)A_{k-1} \geq 2kA_k > 0$ $(k = 1, 2, ...)$, *and if* $x \in (0, 2\pi)$, *then*

$$\sum_{k=0}^{n} A_k \sin((k + \tfrac{1}{4})x) > 0 \quad \textit{and} \quad \sum_{k=0}^{n} A_k \cos((k + \tfrac{1}{4})x) > 0. \tag{1}$$

Inspired by the Askey-Steinig paper we provided a refinement of the cosine inequality given in (1):

$$\sum_{k=0}^{n} A_k \cos((k + \tfrac{1}{4})x) \geq \frac{\alpha A_0 (2\pi - x)^2}{\cos(\frac{1}{4}x)} \quad (0 < x < 2\pi),$$

with $\alpha \approx 0.015$, see Alzer & Kwong [5]. Moreover, we studied (1) with $A_k = 1/(k+1)$ and obtained

$$\sum_{k=0}^{n} \frac{\sin((k + \frac{1}{4})x)}{k+1} + \sum_{k=0}^{n} \frac{\cos((k + \frac{1}{4})x)}{k+1} \geq \frac{1}{\sqrt{2}} \quad (0 \leq x \leq 2\pi)$$

and

$$\frac{5}{8} \cos(\tfrac{1}{4}x) + \sum_{k=1}^{n} \frac{\cos((k + \frac{1}{4})x)}{k+1} \geq 0 \quad (0 < x < 2\pi),$$

$$\frac{5}{8} \sin(\tfrac{1}{4}x) + \sum_{k=1}^{n} \frac{\sin((k + \frac{1}{4})x)}{k+1} \geq 0 \quad (0 < x < 2\pi).$$

The constants $1/\sqrt{2}$ and $5/8$ are sharp. See Alzer & Kwong [6, 7].

From: George Andrews (gea1@psu.edu)
Contributor: George Andrews: grateful beyond words

We are all grateful beyond words to have shared part of our professional lives with you, a grand mathematician and a grand man. This is especially true for me.

I think with fondness on the numerous discussions and collaborations we have had over the years. Also you have greatly inspired everyone around you with the scope and grandeur of your vision. The fact that special functions have come into such prominence owes much to your sustained efforts to promote every aspect of the subject. Decades ago you were pushing hard for a new and much grander version of the Bateman Project. Your leadership inspired many others to get involved and eventually this produced the Digital Library of Mathematical Functions.

I have many times reflected on a conversation we had many years ago in which you referred to someone as "...a very broadminded mathematician: he doesn't care what kind of eigenvalue problem you are working on." You have always epitomized the opposite of that person. You are smart, generous and dedicated to the advancement of mathematics and mathematics education at all levels.

Each of us is saddened by the great difficulties facing you. We love you and are deeply in your debt, especially me.

front row, l to r: Steve Milne, David Bressoud, Mourad Ismail and George Andrews. Second row: Dennis Stanton is behind Mourad.

Dick Askey and George Andrews sightseeing in Tianjin, China in 2004.

Contribution #4 ——— Liber Amicorum ——— Dick Askey

From: Eiichi Bannai (bannai@math.kyushu-u.ac.jp)
Contributor: Eiichi Bannai: a small parallel contribution

Dear Professor Askey: I just received the message below from Paul Terwilliger, through Tatsuro Ito, that you are very sick. I feel very sorry about this.

I feel that I really owe you a lot, beyond what you might imagine. Perhaps you might remember me as a young faculty from Ohio State University who just came from Japan. I was not an expert in orthogonal polynomials, but I really learned the subject through you. Your influence has really broadened my research, and I was able to know many people through you.

I have many things that I would like to say to you, if I could. But at this time, I just would like to express all my thanks. I will never forget you and all the help you have given me.

I was at the Ohio State University in 1972–1974 and 1976–1989. After my retirement at Kyushu University in Japan, I was at Shanghai Jiao Tong University in China in 2011–2017 for 7 years. I now live in Tokyo as I retired, but I am still trying to pursue research in mathematics.

I would also like to say that I am very much honored that I could contribute a paper to your 80th birthday celebration. Also, I am very honored that the case $q = -1$ has recently got the attention of many people as the Bannai-Ito scheme. It is a small parallel contribution to the great Askey-Wilson scheme (owing to Paul Terwilliger, Luc Vinet, and others).

Sincerely yours, Eiichi Bannai

From: Herman Bavinck (hermanbavinck@hotmail.com)
Contributor: Herman Bavinck: fifty years ago

Fifty years ago
Herman Bavinck
September 23, 2019

In 1969 I was employed at the Mathematisch Centrum (now: Centrum Wiskunde & Informatica (CWI)) in Amsterdam and my main duty was to write a doctor's thesis. But I had to find a subject all by myself and so far I had not succeeded.

During the academic year 1969–1970, Dick Askey was spending his sabbatical year at the Mathematisch Centrum. This institute was located in a former school and there was not enough room. In May, my roommate had a terrible accident and it would take months before he could return. Therefore the board decided to give Dick Askey his desk in my room. This was probably the greatest luck I ever had.

When Dick learned about my problem of finding a subject for a thesis, he immediately proposed for me to work with him. He advised me to read Szegő's book 'Orthogonal Polynomials' [125] and many times Dick gave me private lectures at the blackboard in our room. After a while, Dick suggested to me a problem. He could easily have solved this himself, but I managed to solve it. The result was first published as an internal report and later in the Journal of Approximation Theory [41]. Dick refused to be mentioned as a co-author. He said, "*a word of thanks would be enough.*"

Dick gave a series of eight lectures on Orthogonal Polynomials which were attended with great interest by mathematicians from all over the country. These lectures gave me an excellent introduction in this field.

The next problem Dick proposed to me was much more substantial, and a lot of work had to be done. Finally it would lead to my thesis "Jacobi Series and Approximation" (1972) [42]. I was helped by Dick's incredible knowledge of the literature on the subject and his stimulating guidance. When he returned to Madison, we kept in touch by mail and during conferences where we both attended.

When I asked him how I could ever thank him he replied, that the right way was to do the same thing for somebody else. I promised to do so, and during my career at the Delft University of Technology, I always tried to keep this promise.

From: Bruce Berndt (berndt@illinois.edu)
Contributor: Bruce Berndt: myriad of beautiful ideas given to us

When I was a graduate student at the University of Wisconsin, it took me some time to find and begin my journey into analytic number theory. The first courses in number theory that I ever took were both in modular forms, during the spring semester of my third year and the fall semester of my fourth year, with the former taught by Rod Smart and the latter taught by Marvin Knopp. With this background, my research led me to special functions. But, unfortunately, it was too late at the end of my graduate career to have the opportunity to know Richard Askey very well and to take a course in special functions taught by him. It is one of the biggest regrets in my life that I did not take such a course from Dick.

My doctoral thesis featured Bessel functions with a lot of classical complex analysis. Shortly after graduation, Askey approached me with sagacious advice; I will never forget my surprise! I had no idea that he knew anything about me and my thesis! This meeting began his life-long support of my work.

In the approximately 45 years that I have devoted my attention to Ramanujan's (earlier) notebooks [107] and his lost notebook [108], Dick, in innumerable ways, increased my understanding and offered me insights that only he could supply. In each of the Introductions of my five books on Ramanujan's notebooks [44, 45, 46, 47, 48], I express my gratitude to Dick for his careful reading of several chapters, and for his many comments and suggestions. Although I have not personally counted references to each mathematician in my five books, except for Ramanujan, I likely mentioned Askey's name more than any other mathematician.

In recent years, others have referred to me as a member of the "gang of three" for our work on Ramanujan. I could have no greater honor than to be associated in this way with Dick Askey and George Andrews in our quest to understand the myriad of beautiful ideas given to us by one of the greatest mathematicians in the history of our subject.

Bruce Berndt and Dick Askey at the *Askey 80th birthday Conference*, Madison, Wisconsin, USA in December 2013. The picture was taken by Patsy Wang-Iverson.

Contribution #7 ——— Liber Amicorum ——— Dick Askey

From: Gaurav Bhatnagar (bhatnagarg@gmail.com)
Contributor: Gaurav Bhatnagar: thank you, Dick

Thank you, Dick
Gaurav Bhatnagar
September 10, 2019

I first met Professor Askey when my advisor, Steve Milne, introduced me to him over breakfast on the morning of the regional AMS meeting in Richmond in 1994. Since then, I have hung out with him in almost all the conferences that both of us attended. All of us—we friends of Askey—know first hand of his kindness, generosity and good humor. And of course, his inexhaustible fund of interesting stories.

In every meeting, I learnt something interesting, something of everlasting value, of great importance to me personally, of importance to the doing or explaining of mathematics. These are in the nature of what we Indians refer to as the fundas of life. Below, I mention a few things that I have learnt from Dick.

⋆ ⋆ ⋆

What connects us together is Ramanujan. Growing up, I had read a portion of C. P. Snow's foreword to Hardy's Apology, which was about Ramanujan. It was prescribed reading in our high school. From then on, I was immensely interested in anything Ramanujan.

I first heard of Professor Askey in 1987, the year I graduated with my Bachelor's degree from Delhi University. The newspapers reported that three famous professors, Andrews, Askey, and Berndt, had come to India to help celebrate Ramanujan's centenary. Of these three, I had only heard of Andrews, because his number theory book [12] was prescribed by my teacher.

Much later, when pursuing my Ph.D. at Ohio State, under the direction of Steve Milne, I heard of these three professors again. Milne gave a series of courses, mostly following Rainville [106], Gasper and Rahman [77], and Macdonald [94], with additions from Askey's papers, Andrews' "ten lectures" book [11], and from other sources.

At that time a large number of students had gathered around Steve. He made sure that we got adequate exposure to his friends, including these three famous professors. Professor Andrews visited Ohio State for a few days, gave many talks, and advised us on how we should write our paper which was on the topic of inverse relations. Later, Professors Askey and Berndt visited us for a few days in June 1996. Many young people, from Berndt's group in Illinois, as well as from Vienna, came for that mini-conference, and we all benefited from their largesse.

⋆ ⋆ ⋆

I want to present something that to me illustrates the aesthetics of Askey's work. The following is from Math 805, Askey's course on Special Functions at Wisconsin. Shaun Cooper generously shared these notes with me. It is an extract from the first chapter on Legendre polynomials, denoted by $P_n(x)$ below, and requires the generating function of Legendre polynomials. It will be obvious from the context what the generating function is.

It is in Dick's voice, as he wrote it.

⋆ ⋆ ⋆

Subsection 1: From Chapter 1, Math 805, by Dick Askey, circa 1990

A very important fact about Legendre polynomials is that they are orthogonal. One way to prove that is due to Legendre. He computed

$$\int_{-1}^{1} \frac{\mathsf{d}x}{\sqrt{1-2xr+r^2}\sqrt{1-2xs+s^2}} = \sum_{k,m=0}^{\infty} r^k s^m \int_{-1}^{1} P_k(x)P_m(x)\,\mathsf{d}x.$$

The integral on the left can be evaluated as an indefinite integral. I remember teaching this in undergraduate calculus. I bet you never thought you would have to do such an integral in a graduate course. Do it, so that you will really appreciate the following argument due to Hermite.

Consider the integral

$$\int_{-1}^{1} \frac{x^k}{\sqrt{1-2xr+r^2}}\,\mathsf{d}x = \sum_{n=0}^{\infty} r^n \int_{-1}^{1} P_n(x)x^k\,\mathsf{d}x$$

and make the change of variable

$$\left(1-2xr+r^2\right)^{1/2} = 1-ry$$

so that

$$x = y + \frac{r}{2}(1-y^2)$$

and

$$\frac{-r\ \mathsf{d}x}{\sqrt{1-2xr+r^2}} = -r\ \mathsf{d}y.$$

Hence,

$$\int_{-1}^{1} \frac{x^k}{\sqrt{1-2xr+r^2}}\,\mathrm{d}x = \int_{-1}^{1} \left(y + \frac{r}{2}(1-y^2)\right)^k \mathrm{d}y$$
$$= \sum_{j=0}^{k} \binom{k}{j} \frac{r^j}{2^j} \int_{-1}^{1} y^{k-j}(1-y^2)^j \,\mathrm{d}y.$$

This implies

$$\int_{-1}^{1} P_n(x) x^k \,\mathrm{d}x = 0 \text{ for } n = k+1, k+2, \ldots,$$

or equivalently, the integral is zero for $k = 0, 1, 2, \ldots, n-1$, where n is any positive integer. Since $P_m(x)$ is a polynomial of degree m, and so has form $\sum_{k=0}^{m} a_k x^k$, this implies that

$$\int_{-1}^{1} P_n(x) P_m(x)\,\mathrm{d}x = 0 \text{ for } m < n,$$

and by symmetry for $m \neq n$.

⋆ ⋆ ⋆

I presented Legendre polynomials from Askey's lecture notes to Michael Schlosser's Special Functions class in the University of Vienna, when he asked me to substitute for him. After I presented this fragment, I wrote on the board:

> *The supreme art of war is to subdue the enemy without fighting.*
>
> $\sim$ Sun Tzu, *The Art of War*

Many of Dick's own proofs have this quality.

⋆ ⋆ ⋆

There is no one like Dick where it comes to providing motivation. George Pólya once said "*Beauty in mathematics is to see the truth without effort.*" Dick takes a lot of trouble to ensure that you see the truth without effort. Everything in his books, his papers, his lectures, his comments about Euler or Ramanujan, they are all presented in a way that is simple and motivated.

If I want to understand why something is the way it is, I look at Askey.

⋆ ⋆ ⋆

Once Dick told me (as I remember it): "*All I ever did was simple stuff.*". Now, whenever I look at something unfamiliar, it appears far from simple. But what I understand from Dick's words is that if one thinks about an idea enough, it will perhaps begin to appear simple. This even applies to ideas that I think I may have had myself.

Among the many fundas of life given to me by Dick, I think this has been most useful.

Now when I really understand something, understand it well enough to consider it simple; well enough to be able to explain it to others simply—on the blackboard, and on a sheet of paper—then I think of Dick. Thank you, Dick.

From: Nicholas H. Bingham (n.bingham@imperial.ac.uk)
Contributor: Nick Bingham: I just wrote a paper on Gaussian processes—whatever they are

I first met Dick when I visited the University of Wisconsin to give a probability seminar (at the invitation of Peter Ney as I recall) during 1974–75, my year at the University of Michigan. I loved the visit, and Dick was one of a number of people I met there and talked math(s) to. Walter and Mary Ellen Rudin kindly entertained me to a party, where one was able to talk more widely. I consulted Dick about all sorts of things in the interface between special functions (his speciality) and probability (mine). One thing led to another, and by April of the following year (which I spent at the University of Illinois) we had a joint paper, 'Gaussian processes on compact symmetric spaces' [25]. Dick enjoyed dining out on the story: "*I've just written a paper on Gaussian processes with Bingham—whatever they are*".

In the summer of 1976, I went on the 'grand tour', and finished by driving back from Vancouver to Champaign-Urbana, taking good care to stop in Madison to consult Dick about a long paper I was finalizing on Tauberian theorems for Jacobi series [53] (Dick's help is gratefully acknowledged). It was lovely to be able to return his hospitality in London some years later. And what a triumph, that his contributions proved crucial in de Branges's famous proof of the Bieberbach conjecture. That put a stop to patronising comments about special functions! For this relief much thanks, as Shakespeare put it.

It has been a privilege to know Dick, and an even greater privilege to work with him.

Nick Bingham.

From: Ron Boisvert (boisvert@nist.gov)
Contributor: Ron Boisvert: some photos related to the DLMF

Photo taken at the first DLMF [59] Editorial Board meeting at NIST in January 2000. Back row (l to r): Dick Askey, Peter Paule, Charles Clark, Nico Temme, Len Maximon, Michael Berry, Bill Reinhardt, Morris Newman. Front row: Ingram Olkin, Ron Boisvert, Dan Lozier, Frank Olver, Jet Wimp, and Walter Gautschi.

The second photo is from a working meeting of the DLMF Associate Editors at the IMA at the University of Minnesota in July 2002. Dick is at the bottom right, with his back to us, along with Frank Olver. Clockwise from bottom left: Frank Olver, Charles Clark, Ingram Olkin, Michael Berry, Walter Gautschi, Dan Lozier, Nico Temme, Bill Reinhardt, Peter Paule, and Dick Askey.

Contribution #10 ——— Liber Amicorum ——— Dick Askey

From: Patsy Wang-Iverson (pwangiverson@gmail.com)
Contributor: Larry Braden: changed my entire life in one instant

My name is Larry Braden. I had spent years teaching very smart students in private schools in Hawaii, in Russia, and currently in New Hampshire. It was Dick Askey who picked me up from despair in Seattle's Mathfest and changed my entire life in one instant. I mean changed my entire life!

We had just listened to a very clever and glib lecturer spout things like "*Don't be a sage on the stage. A mathematics teacher should be a guide on the side.*" Dick got up in the middle of the auditorium and savaged this guy. Absolutely, very logically, tore his arguments apart. Wow. I thought I was the only one who despised the 1989 NCTM Math Standards. Here was a world-famous research mathematician who did too! I observed someone else in the audience standing and clapping for Dick. I went over and told him how much I enjoyed Dick's attack. "*Go over there and tell him then.*" This person was George Andrews. I followed Dick outside and after a few minutes he invited me to join a group of like-minded people in a group called MathEd.

I laughed with joy and relief! I had felt myself very much like the prophet Elijah, running for his life from Ahab and Jezebel. A voice crying alone in the wilderness, against the travesties of the NCTM. All alone! But no! *Not alone at all!* At the moment Elijah was told "*But seven thousand Israelites have refused to worship Baal, and they will live.*" so Dick assured me "*We are going to fight this nonsense and we are going to win!*"

I cannot tell you how Dick has improved my entire life over the years. Not only with his quiet wisdom, but with the wonderful people I have come to know through MathEd.

Thank you Dick! From the bottom of my heart, I thank you!

Contribution #11 ——— Liber Amicorum ——— Dick Askey

From: David Bressoud (bressoud@macalester.edu)
Contributor: David Bressoud: to come hear my 10-minute talk on the Rogers identities

Dear Dick,

I don't know if I've ever adequately explained how very much you and your example have meant to me over the years. I still remember vividly, and often share, the importance of that afternoon in Toronto when you ignored the announcement of the proof of the four-color theorem to come hear my 10-minute talk on the Rogers identities. The connection you set up between George and Emil Grosswald gave me an opportunity I never would have had otherwise. The year and a half I spent in Madison with you constituted some of the most influential and productive time of my career. You showed me how to think about mathematics. Everything I have written since then, whether research, textbooks, or reflections on educational issues, has been shaped by what I learned from you.

We haven't always agreed, but I have always taken your positions and critiques very seriously, carefully reconsidering my own positions when they have come into conflict with yours.

I am so sorry to hear that you are so ill.
David

From: Richard Brualdi (brualdi@math.wisc.edu)
Contributor: Richard Brualdi: welcome at the *80th Birthday Conference of Dick Askey*

It is my privilege to welcome you, on behalf of the Wisconsin Mathematics Department, to this conference in honor of Dick Askey in the year of his 80th birthday.

Dick arrived in Madison in 1963, two years before I did. I have had many brilliant colleagues here who excelled in research, teaching, and service to the university and to the mathematical community, but only one colleague who was once described as a *national treasure*, maybe by someone in the audience. That description fits Dick perfectly. There is no more dedicated, knowledgeable, unselfish, and tireless mathematician and educator than Dick.

He is a member of the National Academy of Sciences (elected in 1999), an Honorary Fellow of the Indian Academy of Sciences (elected in 1988), Vice-President of the American Mathematical Society (1986–88), MAA Hedrick Lecturer (1996), Turán Memorial Lecturer (Budapest 1991), CBMS Lecturer (1974), ..., I could go on. In Madison, he was named Gabor Szegő Professor of Mathematics in 1986, and also was awarded a John Bascom Professorship in 1995. Dick had 13 PhD students, but his influence goes far beyond his PhD students.

As I expect all of you know, Dick has been tireless in his advocacy of quality mathematics education at all levels. Through his teaching in Madison from Calculus to Special Functions, he has had a profound influence on many people, including I am sure some of you in the audience. One student who had Dick for two semesters of calculus, once sent Dick some quotes he had compiled during the courses. Some of these Askey quotes were

- *I remember more than I should. (Those of you who know Dick are quite aware of his prodigious knowledge and memory.)*
- *In real-life there is no back-of-the-book.*
- *A technique is a trick that works twice.*
- *I don't always tell you the whole truth for which, I am sure, you are eternally grateful.*

This student remarked on how much she had learned from Dick and how much he had influenced her.

Dick retired from the university in 2003 after 40 years as a faculty member. I am sure he had many opportunities to leave Madison but his loyalty to Wisconsin kept him here, and for that we are grateful. While Dick is retired from the university he is by no means retired in the traditional sense. He continues to work hard on education in mathematics with great passion.

So again I welcome you to Madison, turned frigid just in time for this conference, and wish you a wonderful meeting.

Richard A. Brualdi
December 4, 2013

From: William Yong-Chuan "Bill" Chen (chen@nankai.edu.cn)
Contributor: Bill Y. C. Chen: gratitude to you for your encouragement and inspiration

Dear Dick: I would like to express my gratitude to you for your encouragement and inspiration, and would like to convey my best wishes for a recovery to good health.

I felt proud and fortunate to receive your detailed comments on a manuscript of mine, which was truly helpful for the publication of the article.

Thanks a lot for coming to Tianjin twice. Your first trip was for the conference in honor of Jim Louck's 75th birthday in 2004. Jim felt greatly much honored that you could come. He also mentioned to me many times how the Askey-Wilson polynomials came up in physics. Of course, he knew that I did not understand what he was talking about. Gian-Carlo Rota told me that the work of Jim was a gold mine to explore. He asked Jim Louck whether he could give mathematical definitions. Jim always ended up with stories. I wondered whether Jim and his colleagues could understand each other, Jim replied, "*Yes, of course*". So I realized that I should never expect a good answer from Jim, because this was not a good question. I also realized that Jim was really speaking a foreign language. To understand what they do and what they say, I must learn their language and follow their notation. When in Rome, you should probably speak the language as the Romans do. I now felt urged to come back to the unfinished projects with Jim Louck in the hope of a better understanding of special functions and symmetric functions.

Your second trip was for the *Combinatorics of q-Series and Partitions* conference in Honor of George Andrew's 75th Birthday in 2013. I still remember the two non-mathematical comments in your speech. You said that I was a professional to make people work. I have been taking it as a great compliment. But I had a second thought recently. I realized that I had better confine myself to be a mathematician. Being a mathematician, not an administrator, I was not supposed to make people work, while I tried hard to do so. It would be more rewarding to just make myself work. In some sense, I felt that one cannot make other people work, or even persuade other people to work. Our Chairman Mao said that the revolution is not a dinner invitation. Life would be much simpler and easier if I were not in a position to make people work.

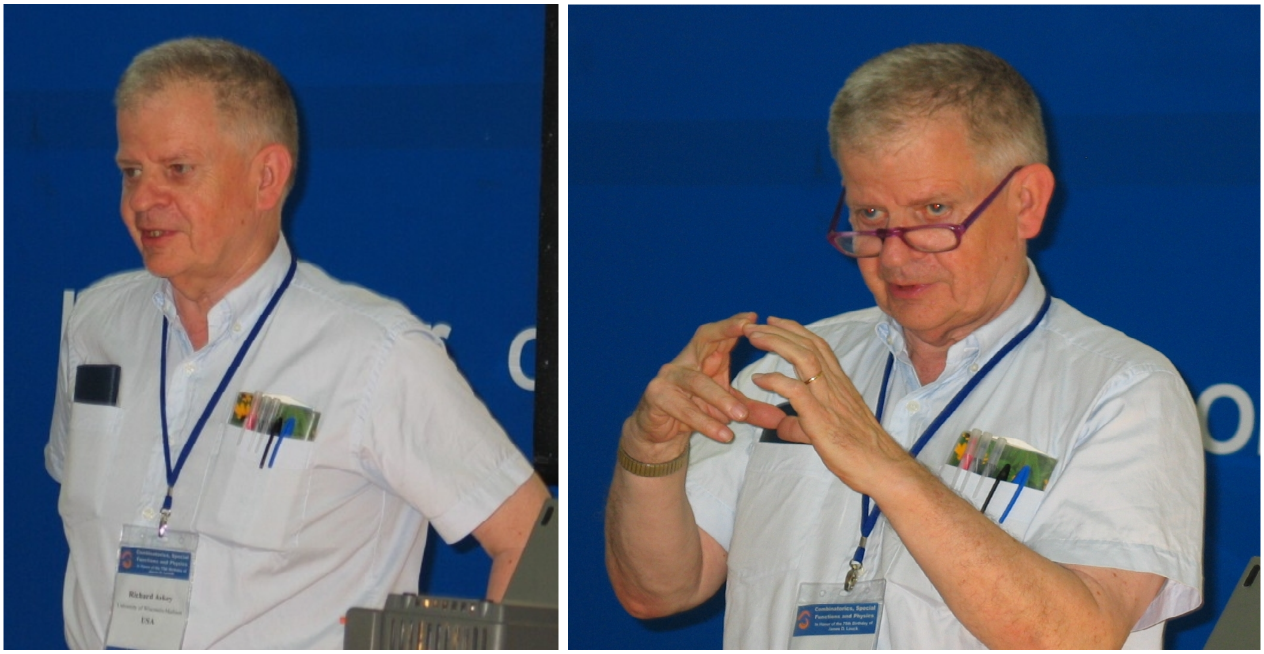
Your second comment was concerned with no supply of toilet paper in China, with only a few exceptions. This was embarrassing but true. Mrs. S.S. Chern once said that it was her wish that someday one could find the bathroom only by the sign without the aid of the smell. Now I would like to report to you that we have made significant progress in this direction. In some places (for example, at the Center for Applied Mathematics of Tianjin University), we have high quality facilities of international standard (in fact, we use "American Standard" products, presumably made in China) with official supply of paper and liquid soap. This was indeed a big step forward. As I get older, I have become more optimistic. Sometimes you cannot change the world and you cannot change yourself, but there may be some hope that you will see the world change itself.

China even launched a toilet revolution in the countryside. I heard a story that in some areas people were not happy with the improvements. The reason was simple: the pipes got frozen in the winter.

It looks that the q-disease has been spreading in China. We are deeply indebted to you for your monumental contributions to and profound impact on the subject that we all love so much.

Take care! Bill, Center for Applied Mathematics, Tianjin University, Tianjin 300072, P. R. China

Dick Askey and Bill Chen at the *Combinatorics, Special Functions and Physics* Conference in Honor of the 75th Birthday of Jim Louck in Tianjin, China in 2004.



Dick Askey speaking at the *Combinatorics, Special Functions and Physics* Conference in Honor of the 75th Birthday of Jim Louck in Tianjin, China in 2004.

Dick Askey in Tianjin, China in 2013.

Bill Chen, Krishna Alladi and Dick Askey at Tianjin, China in 2013.

Dick Askey speaking at the *Combinatorics of q-Series and Partitions* conference in Honor of George Andrew's 75th Birthday at Tianjin, China in 2013.

George Andrews, Dick Askey and Peter Paule at Tianjin, China in 2013.

Dick Askey at Tianjin, China in 2013.

Dick Askey and Bill Chen at Tianjin, China in 2013.

Dick Askey, Dennis Stanton and George Andrews at Tianjin, China in 2013.

Dick Askey and Mourad Ismail at Tianjin, China in 2013.

From: Yang Chen (chenyayang57@gmail.com)
Contributor: Yang Chen: encyclopedic knowledge, deep insight, generosity

Years ago, I was visiting the University of Florida, Gainesville, working with K. A. Muttalib on some q-related problems where the weights decay slowly to zero like $\exp(-a(\ln x)^2)$, $a > 0$, as x gets large. We were looking at Hermitian ensembles which under-pinned transport in disordered systems. We needed the theory of orthogonal polynomials in one variable. It seemed that none of the "classical orthogonal polynomials" would do the job. While, looking around the library, we found the *Bowdoin Conference Proceedings* [65, 66]. In there, we found lectures by Pierre Van Moerbeke, Dick Askey, and George Andrews. There was a lot of q-related material.

I called the Wisconsin, Mathematics Department and they told me Dick needed to have a minor operation. So we called George Andrews and George told us "*Since you are in Florida, why not call Mourad.*" So this is how I got into the subject.

In a meeting at Delaware (where I met Dunkl for the first time), I saw Dick again, who told me to read Widom and Wilf [132] on the least eigenvalues of large Hankel matrices. This led Christian Berg, Mourad and I to come up with a criterion on the indeterminacy of the classical moment problem. My students, and I (mainly Chinese, now that I am in Macau) continue to work on such problems.

Encyclopedic knowledge, deep insight, generosity, to me characterizes Dick as a Mathematician and man.

Dick Askey, Yang Chen and Mourad Ismail at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013.
The picture was taken by Patsy Wang-Iverson.

Contribution #15 ——— Liber Amicorum ——— Dick Askey

From: Ted Chihara (chihara@pnw.edu)
Contributor: Ted Chihara: the world of OP needs old dinosaurs like you and me

I understand you have been having health problems. Please hang in there. The world of OP needs old dinosaurs like you and me: people that still have to use chalk and a blackboard to make a presentation! We missed you in Baltimore and Linz.

Contribution #16 ——— Liber Amicorum ——— Dick Askey

From: Jacob Stordal Christiansen (stordaljc@gmail.com)
Contributor: Jacob Stordal Christiansen: maybe one day you can help a young person

I was visiting Madison in 2002, the year before Dick retired, and had the pleasure of following his course on Special Functions. To the outsider, or the beginner, the area of special functions can seem like a big collection of formulas. But in Dick's world, there is a story behind every formula and the motivation is always clear. This makes the theory really, really beautiful!

I tried to take careful notes and hope that part of Dick's approach is passed on when I myself teach special functions following his book (with Andrews and Roy) [14]. I have never before nor after seen anybody compute as fast on the board as Dick. I cannot help wondering how fast he was years earlier.

I had many fruitful mathematical conversations with Dick. This not only helped me a great deal in my work, but often provided me with a whole new perspective. Dick's view on teaching has also served as a guideline for me for many years. At some point, I asked Dick how I could thank him. His answer—that I'll never forget—was simply: "*Maybe one day you can help a young person.*"

Dear Dick, thank you for everything! You have been and will always be a huge inspiration to me and many others. I admire you as a mathematician, as a teacher, and as a person.

From: Howard Cohl (Howard.Cohl@nist.gov)
Contributor: Howard Cohl: among the most important facts known about these functions

I first came across Dick Askey, in the research library, on the carpeted floor in the stacks, when I was a graduate student in Astrophysics at Louisiana State University (LSU) in Baton Rouge, Louisiana. My Physics PhD thesis was focused on obtaining the Newtonian potential for isolated self-gravitating systems. I therefore needed to study the behavior of the $1/r$ potential on $\mathbb{R}^3$, and in particular in cylindrical coordinates.

The $1/r$ potential, the potential due to a point charge/mass at the origin, evaluated at a measurement point, satisfies Laplace's equation everywhere except within a singular region where the two points coincide. I was eager to learn everything I could about separable solutions to Laplace's equation, especially in rotationally-invariant coordinate systems, all of which were described beautifully in a book by Willard Miller, Jr., "Symmetry and Separation of Variables" [97]. I had recently discovered that a reciprocal square root identity due to Eduard Heine [80], governs the expansions of the $1/r$ potential in all rotationally-invariant coordinate systems. This was an expansion in terms of Chebyshev polynomials of the first kind, namely

$$\frac{1}{\sqrt{z-x}} = \frac{\sqrt{2}}{\pi}\sum_{n=0}^{\infty} \epsilon_n T_n(x) Q_{n-\frac{1}{2}}(z), \tag{2}$$

where $\epsilon_n = 2 - \delta_{n,0}$, and $Q_{n-\frac{1}{2}}$ is the associated Legendre function of the second kind, a toroidal harmonic in fact. This expansion implies that in every rotationally-invariant coordinate system which separates Laplace's equation, you get a free addition theorem for the azimuthal Fourier coefficients. Applied in spherical coordinates, one obtains a second addition theorem for spherical harmonics, one which sums over all meridional modes for a fixed azimuthal mode.

Sitting on the floor of the library at LSU, scanning Miller's book, I happened across the book's Foreword, written by the *General Editor* for the Section on Special Functions, *Richard Askey*. By this point in my education I had realized that I was fascinated by special functions, and as I started to read the Foreword of Miller's book, I was engrossed. From everything I had learned up to that point, it was clear that this was a magnificent introduction to special functions. The study of (2) had led me to a new addition theorem for spherical harmonics, and right in the middle of Dick's introduction to special functions, was the statement that the addition formulas for Legendre polynomials (spherical harmonics) and Chebyshev polynomials of the first kind (circular harmonics), *are among the most important facts known about these functions.* I then came to the realization that I had dug into the very heart of the functions I was studying and had learned something fundamental about them. I recognized that I must be on the right track in the exploration of the unknown properties of the wonderful patterns within equations which in some sense, govern the behavior of the special solutions to linear partial differential equations which govern the behavior of the universe.

From that point onwards, everything that I learned from Dick, from reading his papers and books, his phenomenal monograph [19], and from personal communications that I had with him, captured the beauty of special functions and orthogonal polynomials. There's still so much to learn. Thank you for inspiring those around you with the beauty of truth, that you are able to so clearly see.

Aashita Kesarwani, George Andrews, Howard Cohl, Dick Askey and Kathy Driver at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013.

From: William Connett (connettw@gmail.com)
Contributor: William Connett: the torrent of information that came out of the phone

Dick Askey in St. Louis
William Connett

Dick Askey grew up in St. Ann, Missouri, a suburb of Saint Louis, not far from the location chosen by the University of Missouri for its St. Louis campus (UMSL). The new campus was founded in 1965. Although I am pretty sure that Dick had nothing to do with the selection of the location of the new campus, he was always a special friend and advisor to the developing Mathematics Department at UMSL. He had befriended our longtime chairman, Debby Haimo, at Washington University and Harvard, and had introduced Alan Schwartz to the joys of Special Functions in his famous course on this topic that he taught at The University of Wisconsin, Madison. He also gave numerous colloquia here at UMSL, and served in a variety of capacities (tenure committees, program evaluation committees, etc.) in our struggle to build a successful research department.

I met Dick at the University of Chicago when I discovered that we had similar tastes in mathematics. In those days, each library book contained a card behind the jacket which indicated who had taken out this volume, and when. Much to my surprise, I discovered that every book or journal that was published before 1900 that I looked at had Dick's signature on the card. Who was this guy? It took me a while to find out.

This happened at UMSL when Alan Schwartz and I were working on a collection of problems in the 1970s and 1980s that required intimate knowledge of various families of special functions (for example the Jacobi polynomials or the spheroidal wave functions). Alan and I would work away at the problem until we were stuck, and then we would call Dick. He was always generous and helpful, but our problem was trying to keep up with the torrent of information that came out of the phone from Dick's end. Alan and Dick would be on one phone talking and I would be on a second phone writing as fast as I could. At the end of the call we were emotionally and intellectually exhausted, but we would have eight or ten pages of suggestions and references to check out. What a gift. Alan and I would then spend the next couple of weeks sorting through my chicken scratch, and almost always came up with another way to attack, and frequently to solve our problem.

Not only was Dick a superb mathematician, he was the champion of a whole world of mathematics that had fallen out of fashion. He and his students and collaborators brought these areas to life again, and in the process made fundamental contributions not only to Special Functions, but to problems in areas which seemed far away, for example, number theory, combinatorics, statistics, and physics.

There is a famous Pub in the Delmar Loop called "Blueberry Hill" owned by an entrepreneur named Joe Edwards. One of Joe's projects is to honor famous St. Louis natives with plaques containing brass stars displayed on the sidewalks in the Loop. For example, Chuck Berry, Howard Nemerov, T. S. Elliott, Tennessee Williams, Maya Angelou, William Burroughs, Scott Joplin, Kevin Kline, Vincent Price, and many others are in the "Delmar Walk of Fame". I seriously believe that Dick is deserving of a commemorative brass star as well. Next time I see Joe, I will make the suggestion.

William Connett,
Department of Mathematics, University of Missouri–St. Louis, St. Louis MO 63121

From: Shaun Cooper (S.Cooper@massey.ac.nz)
Contributor: Shaun Cooper: it was as if Dick had direct access to Riemann

I met Dick Askey when I was a first-year graduate student at the University of Wisconsin. Having seen the advertisement for the weekly "Special Functions Seminar", I was intrigued and went along. When Dick noticed I was new, he asked my name and I was immediately made to feel welcome. The topic that semester was Epstein's zeta function, following Selberg and Chowla [116], and the next 50 minutes I can only describe as thrilling. I had never seen anything like it in terms of the insights and ideas that were revealed—I can remember thinking it was as if Dick had direct access to Riemann—and my brain had to work as fast as it could to try to keep up. At the end of my first year I passed my qualifying exams and Dick agreed to take me on as a student.

The Special Functions Seminar continued for every semester, and quite often Dick also taught a graduate course in Special Functions Math 805 that I also enrolled for. The other regular participants in the seminar were Ranjan Roy and fellow student Warren Johnson, both of whom gave clear and beautiful lectures. Many other mathematicians, especially visitors, joined the seminar from time to time. The seminar remains the best learning experience in all of my education at any level.

During the seminars, Dick would frequently mention unsolved problems. I solved one of them, something about a constant term in a Macdonald-type identity associated to the root system G_2, and expanded the topic to make it into a thesis.

A frequent subject of the seminar, which I just loved, was the mathematics of Srinivasa Ramanujan. One day, Dick gave me a copy of the monograph "Development of Elliptic Functions according to Ramanujan" that had just been written by K. Venkatachaliengar and suggested I edit it. The typographical errors were not too difficult to fix, but there were some other issues that were deeper, for which I lacked the skills and knowledge to correct. I kept coming back to the monograph over the years and I am delighted that the edited book was published by World Scientific in 2012 [127]. That makes it the most overdue homework assignment I have completed by about 20 years!

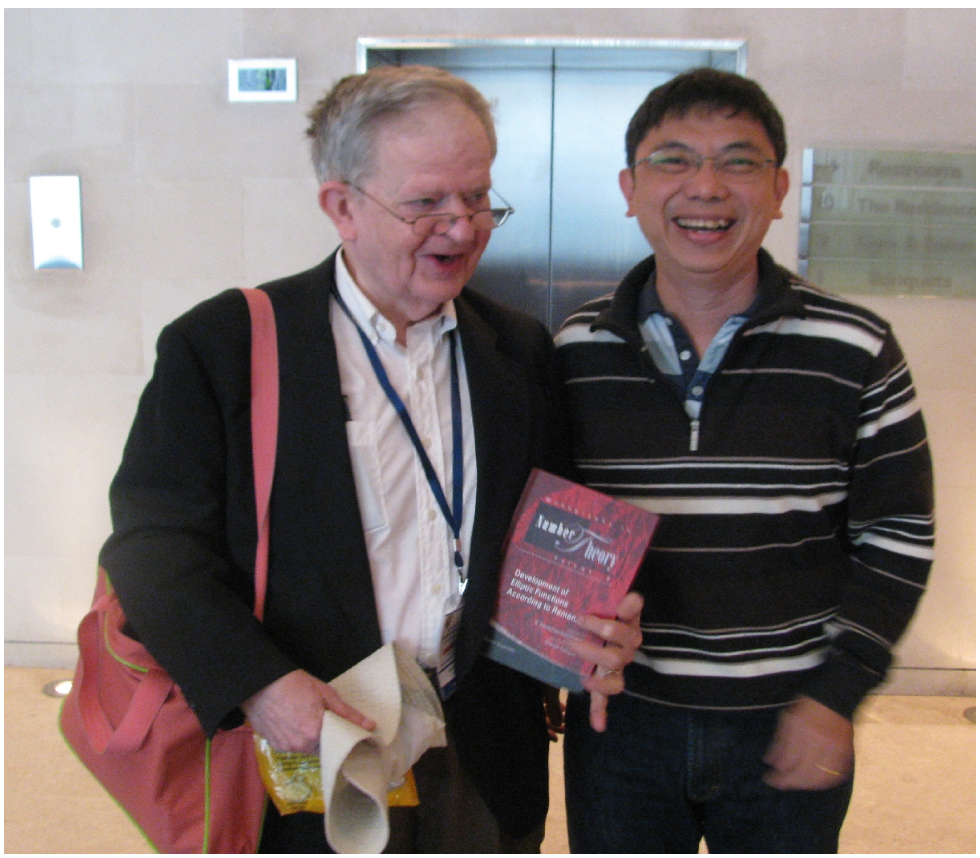
One of the fun things about Dick's teaching was his informal definitions. His definition of "*to understand something*" for an undergraduate student, was "*to be able to explain it to a younger brother or sister in a way that they would get it.*". The definition he applied for himself, and that he encouraged graduate students to use, was "*to be able to give a lecture on it, in a year's time, without notes or preparation.*"

Dick expressed great personal interest in my progress. He made sure that I met mathematicians from other universities and countries and made it possible for me to attend several conferences. After I graduated and returned to work in New Zealand he was concerned that I did not become isolated and helped me make contacts that have been essential in my career development and for which I will always be grateful.

In conclusion, thank you Dick for everything. I learned so much.

Shaun Cooper

Dick Askey and Shaun Cooper at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013. Photo taken by Patsy Wang-Iverson.



Dick Askey and Heng Huat Chan at Ramanujan 125 at Delhi University, Delhi, India in December 2012. "*Dick was holding Cooper's book and he was obviously very proud of Cooper.*"

From: Persi Diaconis (diaconis@math.stanford.edu)
Contributor: Persi Diaconis: what my old colleagues Pólya, Szegő, and Dick Askey had to say!

Dear Friend Dick,

You are one of my heroes. Not just because of your wonderful work but because of your bravery under fire. As we both know, there was a long time when our math world just didn't know what to think about orthogonal polynomials: was it applied math, a corner of representation theory, or numerical analysis? Just what was it??

Anyway, it got "no respect". You kept soldiering on and beat the bastards at their own game. I recently taught an orthogonal polynomials course here at Stanford. I had 20+ grad students take it for credit—and quite a few of them really learned something about what my old colleagues Pólya, Szegő, and Dick Askey had to say!

Your steadfastness through 50 years of scrutiny, with humor and good taste, has really meant the world to those of us in your orbit.

All good things,
Persi Diaconis

Persi Diaconis and Dick Askey at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013. Photo taken by Patsy Wang-Iverson.

From: Karl Dilcher (Karl.Dilcher@Dal.Ca)
Contributor: Karl Dilcher: one of those rare pivotal moments in my career

Dear Dick,

We've seen each other on and off since 1985 or 1986, and talking with you was always particularly refreshing and inspiring.

One early encounter still sticks in my mind, with great gratitude. In the Summer of 1986, at the time of a large Approximation Theory conference in Edmonton, Alberta, I had recently submitted one of my first non-thesis related papers to the SIAM J. of Math. Analysis, of which you were an editor (if not Editor-in-Chief).

During a break at the conference you took me aside, pulled a copy of my paper out of your briefcase, and went over it with me. On one level, this was an extremely kind gesture by an editor towards a young and beginning author, totally in line with what everybody has written about you.

But for me personally, it was one of those rare pivotal moments in my career. Here was this famous mathematician who had just given a plenary talk about his (and George Gasper's) essential contribution to the proof of the Bieberbach conjecture—and he took the time to talk with a young postdoc from far-off Halifax, taking his work so seriously. This was an enormous boost at a crucial time when I wasn't sure yet where I was headed professionally. In addition, you gave me valuable hints about good and effective mathematical writing, which I followed in this and all further papers.

Thank you, Dick, for everything.

With all my best wishes,
Karl Dilcher
Dalhousie University

From: Atul Dixit (adixit@iitgn.ac.in)
Contributor: Atul Dixit: lucky enough to have witnessed one such

The first time I heard about Professor Askey was in Professor Roger Barnard's 'Special Functions' course at Texas Tech where I was doing my Masters. He was full of respect for Professor Askey in view of his everlasting contributions to this field. Also, the first time I got introduced to q-series was when I stumbled upon his paper "Orthogonal polynomials and theta functions" from 1989 [21].

To my surprise, he immediately replied supplying some more details. With renewed interest, I began working on it again only to get stuck at some further step. I hesitatingly again wrote to Professor Askey asking for help. He again replied providing me a further hint. At the end of his email, he wrote – "*if you aren't able to complete the proof, I can call you to discuss the solution. In that case, please give me your phone number*". However, I was able to complete the proof with that second help from him and thankfully he didn't have to call me for it.

That day I witnessed his dedication to help students. It left a lasting impression on me. Back then we hadn't even met in person! I am sure there must be many more students and many more incidents like these. I am lucky enough to have witnessed one such in my life!

Professor Askey, I wish you many, many more years of happiness and good health!

Dick Askey and Atul Dixit at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013.

From: Diego Dominici (diego.dominici@dk-compmath.jku.at)
Contributor: Diego Dominici: how Dick Askey taught me to add

How Dick Askey taught me to add
Diego Dominici

I gave my first talk in the field of orthogonal polynomials at the 2004 AMS annual meeting in Phoenix, AZ. The room was very dark, so it was impossible to see the audience. At the end of my presentation, a gentleman with a very deep voice asked me a question. I carefully explained to him some details, believing that he was not familiar with the topic (it was a session on analysis). We kept chatting while walking out of the room, and when I saw him clearly I realized to my dismay that I have been attempting to teach Krawtchouk polynomials to Dick Askey....

I contacted him a year later, inviting him to participate at a session I was organizing, and he politely refused saying:

> Dear Professor Dominici,
>
> I have been spending almost all of my time on mathematics education, so have nothing new to talk about, and little time for meetings since math education takes a lot of time.
>
> Best wishes on a successful meeting.
>
> Sincerely,
> Dick Askey

I had better luck in 2009, when I asked Dick if he would be willing to be the plenary speaker at the *Spuyten Duyvil Undergraduate Mathematics Conference* that I was organizing. This time he happily accepted. He sent me a title and abstract for his talk:

> Dear Diego,
>
> Please redo this so the formulas look nice. And enjoy playing with the sums mentioned. Dick
>
> Title: Mathematics from the comic strip Fox Trot
>
> Abstract: One place one never expects to find interesting mathematics is a comic strip. However, the following problem appeared in Bill Amend's Fox Trot about ten years ago
>
> $$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^2}{k^3+1} = \quad ? \tag{3}$$
>
> With the exception of one result of Euler, which will be described, all that is needed to find the sum of this series is taught in calculus. However, some of the tools we will use are not used this way in calculus. You know about changing variables in an integral, but have you ever thought about changing variables in a sum? When you start to think about it, it seems silly. Yet, it can be useful, not only to help find the sum of the series above, but to do some surprising and important mathematics. One example is how to find the sum of a series which is an approximation to the normal integral, the integral on the real line of e^{-x^2}.

Dick Askey with undergraduates at the *Spuyten Duyvil Undergraduate Mathematics Conference* at New Paltz, New York, USA in 2009.

The series (3) is sometimes known as the "Fox Trot Series". Dick gave a wonderful presentation that was well received by the students, even if the mathematical level was way above their heads.

Unfortunately I can't find the notes I took during the talk, so here is a possible way of finding the value of (3). First, use partial fractions

$$\frac{k^2}{k^3+1} = \frac{1}{3}\frac{1}{k+1} + \frac{1}{3}\frac{2k-1}{k^2-k+1}.$$

The first term gives a trivial sum

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k+1} = 1 - \ln 2.$$

For the second, use the Mittag-Leffler expansion of the hyperbolic secant function

$$\pi \operatorname{sech}(\pi z) = 4\sum_{k=0}^{\infty} (-1)^k \frac{2k+1}{(2k+1)^2 + 4z^2} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{2k-1}{k^2-k+z^2+\frac{1}{4}},$$

where $z \neq \pm\left(k+\frac{1}{2}\right)i$. Taking $z = \frac{\sqrt{3}}{2}$, we get

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{2k-1}{k^2-k+1} = \pi \operatorname{sech}\left(\pi\frac{\sqrt{3}}{2}\right),$$

and therefore we obtain the answer given in the wonderful book by Andrews, Askey, and Roy [14, p. 60]

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^2}{k^3+1} = \frac{1}{3}\left[1 - \ln 2 + \pi \operatorname{sech}\left(\pi\frac{\sqrt{3}}{2}\right)\right].$$

Dick was clearly very fond of this problem, because he wrote about it in one of his recent papers [24].

I owe an immense debt of gratitude to Dick Askey for his support, insight, and the joy of sharing time with a wonderful human being.

Thank you Dick!

Contribution #24 ——— Liber Amicorum ——— Dick Askey

From: Kathy Driver (kathy.driver@uct.ac.za)
Contributor: Kathy Driver: you are looking at the zeros of scaled/shifted ultraspherical polynomials

On every occasion that I met Dick, he was affirming and encouraging no matter which problem I bombarded him with. He would sit and discuss insights he had with me and then always refuse to have his name on the paper!

On the occasion of the celebration of Dick's 65$^{\text{th}}$ birthday at Mt. Holyoke, Peter Duren and I approached Dick at his birthday dinner and, after exchanging greetings, we told him we had used recent asymptotic results of Peter Borwein and Weyu Chen to obtain an asymptotic result for zeros of some ${}_2F_1$ polynomials. All worked well, their methods had been successfully applied. The only concern we had was that when we checked numerical plots of the zeros of our class of ${}_2F_1$ polynomials, instead of the pleasing picture of the zeros hurrying towards the curve as n increased, **all** the zeros actually lay **on** the curve for each value of n, which was, in that case, the circle $|z-1|=1$. The zeros were expected to approach the circle, not lie on the circle....

Peter and I explained what we had done and asked Dick if he could offer any insight or reasons. Dick took out a sheet of paper, thought for about 20 seconds, wrote down a formula and said: "*You are looking at the zeros of scaled and shifted ultraspherical polynomials.*".

He was able to do this without telling us that we had not properly done our homework... This anecdotal and amusing story (which wasn't such fun at the time) sums up my many fruitful and wonderful discussions with Dick. It was always about the subject he loved, never about himself. Peter and I did our homework and wrote some papers thanks to Dick.

Dick was not just helpful and interested when approached. He was also positively encouraging and would take the initiative by setting aside times at meetings to talk. I have several "originals" arising from those great discussions with Dick at conferences where he was so generous with his time.

My favourite story about Dick emanates from the first time I met him at an *OPSF conference* in Granada in 1991, the first international conference that I attended. He told me he didn't like the notation I used in my talk. After this admonition to do better next time, he and I then attended a talk where he promptly fell asleep (not quietly, much to the amusement of all), had a wonderful nap for the entire duration of the talk, and the moment it ended and applause subsided, Dick's hand shot up and he asked a penetrating question. I remember thinking Hmmm, this guy really **is** a star....

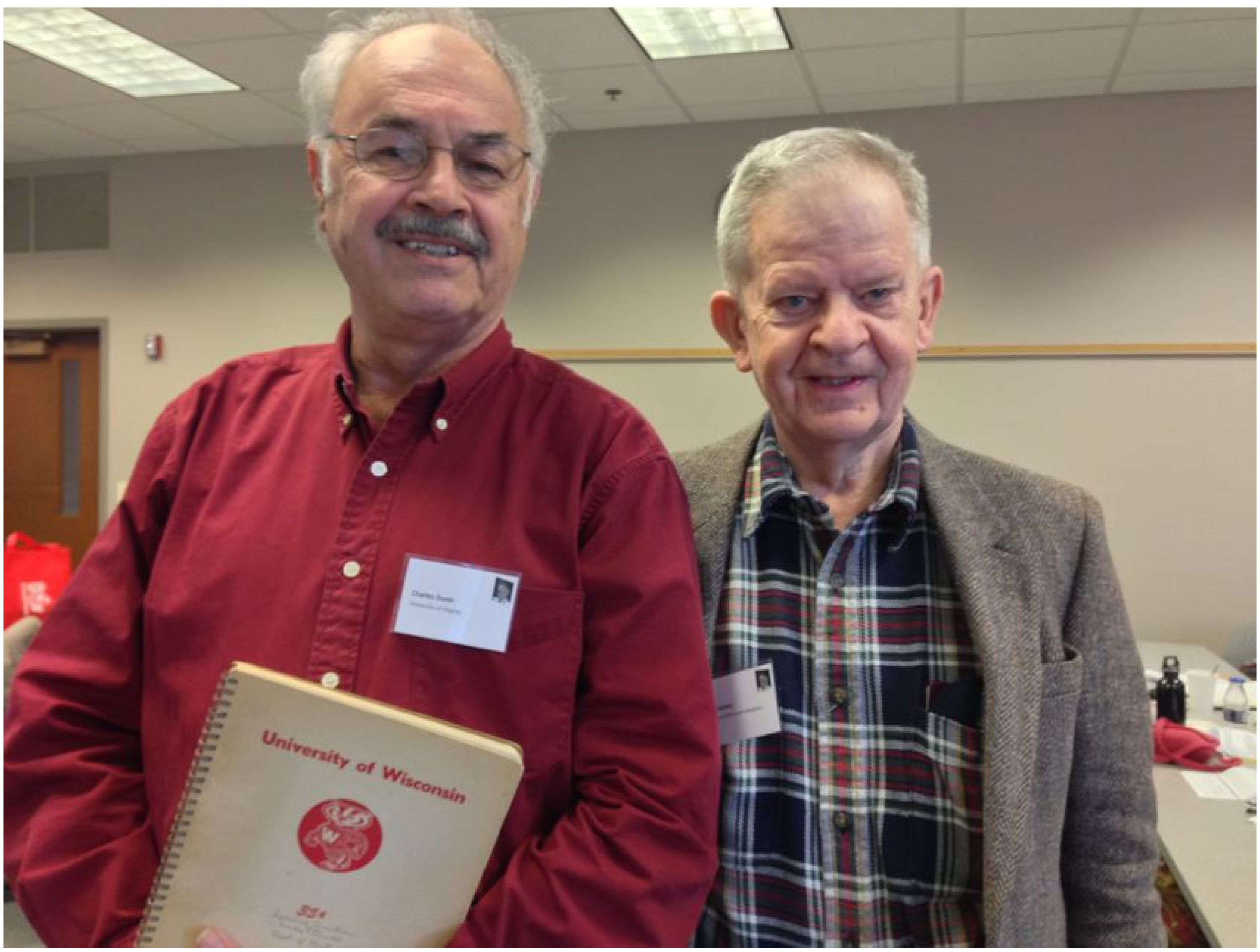
Kathy

Contribution #25 —— Liber Amicorum —— Dick Askey

From: Charles Dunkl (cfd5z@virginia.edu)
Contributor: Charles Dunkl: now I am not even afraid of ${}_5F_4$'s

I am sorry to hear about your health problems. I hope the treatment is going well. Sometimes I am taken aback at how many years have gone by since we started our careers—decades (even I had a conference for my 75$^{\text{th}}$ birthday already). It was 1964 or so when I took your Special functions course. I appreciated the update (or should I call it an upgrade) you gave me on hypergeometric series, when I visited Madison during a time when George Andrews was visiting. Essentially you called me in your office, told me to bring a yellow pad and then gave me a brief but intensive lecture on the various summations. Those pages were really valuable for me—now I am not even afraid of ${}_5F_4$'s. You are probably amused to read about your contributions to the nonassociative algebra crowd—arXiv lists 23 papers on the Askey-Wilson algebra—there are applications to quantum groups, special behavior at roots of unity, higher dimensional versions and so on.

We are all thinking of you,
Best wishes,
Charles



Charles Dunkl and Dick Askey at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013. Photo taken by Patsy Wang-Iverson.

From: Loyal Durand (ldurandiii@comcast.net)
Contributor: Loyal Durand: positivity problems to be explored—fun for a physicist!

I have many happy memories of my interactions with Dick Askey over the years. I first consulted with him in the mid 1960s over a problem involving addition formulas for Gegenbauer functions of the second kind, needed in physics calculations in scattering theory that my postdocs and I were involved with. It turned out that he did not know the kind of result we needed—we went on to derive the necessary results and many more, and I later extended these in various directions. Still, it alerted Dick and me to each other's interests, and I became aware of the "Askey Seminar" on special functions, which I would attend as subjects of interest come up.

In 1969, Dick asked me if I knew how to get an analogue of Nicholson's integral for Bessel functions for the case of Legendre functions. Based on my earlier work, I was able to write down the key integral expression overnight, but there were pieces missing that I did not fill in until 1971 when, stimulated by a series of lectures in his seminar, I went back and derived the Nicholson-type formula for the more general case of Gegenbauer functions. He subsequently alerted me to positivity problems which could be explored with this starting point—fun for a physicist!

Over the following years, Dick alerted me to the work of Tom Koornwinder on Jacobi functions, group theoretic relations, and more. This was the basis for subsequent work of my own on product addition formulas for the functions of the second kind and the related Nicholson-type integrals; and the discovery of a symmetrical addition formula for the Laguerre functions. Later work on fractional group operators and special functions was originally stimulated by an invitation to speak in a special session at an AMS meeting—but not completed for some 25 years. And he kept me aware of other important developments in the area of special functions.

Through all this period, I spoke occasionally—every year or so—in the Askey seminar, usually a few talks on topics of interest in both mathematics and physics—the last was a series of lectures on diagrammatic methods for handling the calculation of transformation formulas in the theory of angular momentum in quantum mechanics, with applications to hypergeometric functions, extendable to other group contexts. Dick also made sure I was aware of other events of interest in the math department, and that I met visitors with overlapping interests. This was all very stimulating and, despite our different backgrounds and ways of thinking, quite productive. It is really nice to think back on all those years of interactions and what came out of them.

From: Harold M. Edwards (edwards@cims.nyu.edu)
Contributor: Harold Edwards: formidable contributions to the mathematics that we all love

As I recollect, the AMS establishment scorned any work having to do with the history of mathematics until the US bicentennial in 1976. In that year, the joint meetings had an unusual special session on the history of mathematics in America. Since that small beginning 43 years ago, the role of history of mathematics in the programs of the joint meetings, as well as in publications and other AMS functions, has grown and grown.

At that time, 43 years ago, the membership of the AMS consisted almost exclusively of research mathematicians, so for these new activities the Society had to call on research mathematicians who were historians as a result of their natural inclinations.

Enter Dick Askey. There was never any doubt that Dick was first and foremost a research mathematician, but his work was informed by an encyclopedic knowledge of previous work on the problems that interested him, and accumulating that knowledge required a lot of historical research—something that was an integral part of his work, but also something that came naturally to him.

As valuable as his historical knowledge was to others of us who valued history, his idiosyncratic nature was just as valuable. I don't know if others see him this way, but to me Dick has always seemed to be someone who has his own take on just about everything, and his take is usually something you wouldn't have thought of yourself, often because it depends on a fact that only Dick knows.

As a result, his contribution to the project of assembling the valuable three volume series "A Century of Mathematics in America," published by the AMS [60, 61, 62] in the wake of this newly developed interest in history, was very special and important. And his connection with this project meant that he was part of the coterie that shaped all of the activities of the Society in history in those early days. His presence at the table was certainly a great asset.

I have mostly known Dick through seeing him at the joint meetings over the years. At one of them—after the historians had established a firm beachhead in the programming of AMS activities—there was a plenary talk on the work of Euler, given by a speaker whose name was not known to me before I went to the talk. I found the talk charming—a real crowd-pleaser—and was delighted to learn of the existence of this new exponent of Euler (in the nonmathematical sense) who could so entertain and engross an audience.

When the talk ended, I joined the group around the podium to thank the speaker for his talk, but Dick got to him first and to my surprise he bluntly told the speaker what had been wrong in his presentation of Euler's work. I remember no details at all, but in retrospect I'm sure that Dick was right and also that he was justified in being blunt. It's one thing to charm and entertain and another thing to convey correct ideas and correct understanding. I have heard subsequent talks and read subsequent publications of that speaker and have found that they were more often objectionable than they were entertaining.

For these reasons and many more, I am happy to join in this tribute and expression of gratitude to Dick Askey, for his vigorous thinking, his energetic defense of what he sees as the truth, and his formidable contributions to the mathematics that we all love.

Contribution #28 ——— Liber Amicorum ——— Dick Askey

From: Ismor Fischer (ifischer@wisc.edu)
Contributor: Ismor Fischer: another feather in your overflowing cap

I received my Ph.D. under Dick in 1989. One of my fondest memories during that time was that, not being a great examination taker, I was experiencing some difficulty passing the Analysis qualifying exam at UW-Madison, but nevertheless approached him about the possibility of being his doctoral student. He had enough faith and willingness to give me a chance to prove myself, by assigning problems for me to work out from Pólya and Szegő. He ultimately became convinced of my capability, and agreed. It would have been easy for him to flatly refuse, and tell me to come back when I passed (which I eventually did, of course). The training, insight, and experience I acquired proved to be invaluable. A colleague of mine in California and I are presently co-authoring a paper that settles a long-standing conjecture, which will be dedicated to him, and would have been impossible without his professional guidance and personal generosity, all those years ago. So if you can manage it Dick, squeeze yet another feather in your already overflowing cap, and many thanks once again.

Contribution #29 ——— Liber Amicorum ——— Dick Askey

From: Dominique Foata (foata@unistra.fr)
Contributor: Dominique Foata: the pope of Special Functions

Dick Askey, known as the pope of Special Functions, has certainly educated a generation of combinatorialists in his discipline. In the mid-seventies, the study of Special Functions had not entered the field of Combinatorics in full force. We had timid combinatorial evaluations in terms of integrals of orthogonal polynomials, but nothing systematic had been made.

I discussed the matter with George Andrews during the Berlin meeting in 1976 and asked him if he could help me to have enough specialists in Special Functions for the next Oberwolfach seminar. His answer was: "*you've got to write to Dick Askey.*", which I did.

A few days later I received a very encouraging letter from Dick in his inimitable imperial style: "*you must invite Joe Gillis* (a wonderful mathematician, Doron Zeilberger's mentor; with some nostalgia I remember the 1986 Weizmann Institute meeting in his honor), *and Chuck Dunkl* (Dick called him Chuck), *and Tom Koornwinder, and Mourad Ismail*, and others. Practically all came and it was a very fascinating rencontre. I must say that the "tableau d'Askey" or the "Askey scheme" was born there. I was sitting next to Curtis Greene and we both were fascinated by that surprising diagramme of polynomials knowledgeably commented by his creator.

Since then, Dick has remained the great scholar who has extended his influence to several fields of mathematics, including Combinatorics.

Dominique Foata (Strasbourg)

Contribution #30 ——— Liber Amicorum ——— Dick Askey

From: Jet (Wimp) Foncannon (bolumanis@comcast.net)
Contributor: Jet (Wimp) Foncannon: many distant and exotic places we have celebrated together

Dick, I am thinking of you, and I appreciate the many long years of professional companionship we have shared, as well as the many distant and exotic places we have celebrated together. I never dreamed as a student at Washington University that we would enjoy a lifetime of similar interests. You have—more than anyone else—made the field of special functions attractive and enduring. Many thanks, and fondest best wishes.

Contribution #31 ——— Liber Amicorum ——— Dick Askey

From: Cyndi Garvan (CGarvan@anest.ufl.edu) and Frank Garvan (fgarvan@ufl.edu)
Contributor: Cyndi & Frank Garvan: only true friends will leave footprints on your heart

We were a young family. Gerard was a baby, Jeff was three, and Mike was six. Frank had finished his PhD and had interviewed for permanent positions at several universities with no luck in getting any job offers. At this time he was teaching at a branch campus of Penn State in York, Pennsylvania. He was resigned to stay there, he was the one we were dependent on to pay the bills. I will then never forget receiving a phone call from Frank. "*Cyndi, I just took a post doc job at University of Wisconsin.*" "*Frank,*" I said, "*In America it is custom to talk to your wife before making that kind of decision.*" "*Oh yeah,*" Frank said, "*but this is Dick Askey!*" He was so excited and happy and of course I would move to any place with him but I did think it worth mentioning that wives are generally involved.

So off we moved to Madison with three little boys and our worldly possessions packed in a U-Haul truck. We had no idea where to live but we did know how to camp. So we camped in Madison until we found a place to rent (our decision was probably a bit hasty but two weeks of camping with a baby and two small boys took its toll on our standards).

And then I got to meet Liz and Dick Askey. They were so kind to us. They invited us on excursions and to parties. The Askeys took us to see a play by Shakespeare at the outdoor American Players theatre. I remember the evening as magical. Being with such an intelligent and interesting couple who were taking time to show Frank and I the treasures of Madison.

Over the years and at many conference occasions, I had the great opportunity of spending time with Liz. We were in rooms pretty close to each other during the *Ramanujan's Centenary conference* at Urbana. Liz kept me sane while I was perpetually chasing my three young sons. She was always calm and upbeat with a fantastic sense of humor. I remember her vast interests in so many things. She took advantage of seeing whatever a locale had to offer (e.g., the special children's book collection at the University of Florida). In short, Liz was an inspiring role model for me. I have always looked up to her.

Dick Askey is one of my heroes for many reasons. He helped Frank's career at a very critical time. He was an amazing mentor to Frank. I can remember example after example of his thoughtfulness of others. I admired his concern about the inability of Freeman Dyson to attend a Florida conference and how he taught conference goers the tradition of sending a note to someone who could not attend. Whenever I spoke to Dick about my messy work in the world of medical research, he would wisely and respectfully remark that, "*Math is easy, People are hard.*"

I am grateful that I got to be a small part of the community that Dick and Liz, Bruce and Helen, and George and Joy built. A community that felt like family whenever we had the opportunity to convene and share the latest mathematical discoveries and just catch up with life's events. The world of mathematics is a world of beauty. The community built by the Askey's, Berndt's, and Andrews' is also a world of kindness.

This is where Cyndi asked me to add my part. Reading Cyndi's part has reminded me what we should remember. The q-world and the Ramanujan world is a very nice place to be and the leadership of Dick Askey, George Andrews, and Bruce Berndt has made it so. Repeating Cyndi I again thank Dick for bringing me to Wisconsin and introducing me to many wonderful aspects of mathematics. Dick encouraged me to go over to Physics Dept and learn the computer algebra software REDUCE. This was the beginning for me in a life of q and experimental math. It was very clever for Dick to get me to referee a paper that was completely outside my experience. This pushed me into the problem of the Macdonald identities and surprisingly led me to tackle the F_4 case etc. Throughout my career signposts have magically appeared and I have trusted my instincts to follow them. In Wisconsin Dick showed me the signpost towards Dennis Stanton and Minnesota. Thanks again Dick for being a wonderful mentor and human being.

Dick, we are praying for your comfort and peace. Eleanor Roosevelt said that, "*Many people will walk in and out of your life. But only true friends will leave footprints on your heart.*" The Askeys have left footprints on the hearts of many of their friends in the world of q and Ramanujan's mathematics. We are all the richer for the presence they have had in our lives.

Cyndi and Frank Garvan, September 9, 2019

l to r: Dick Askey, Krishna Alladi, Frank Garvan, Michael Hirschhorn, at the *Combinatorics of q-Series and Partitions* conference in Honor of George Andrew's 75^{th} Birthday at Tianjin, China in 2013.

From: George Gasper (georgeg012@gmail.com)
Contributor: George Gasper: thinking of you and thanking you

Thinking of you and thanking you

George Gasper

My first correspondences with Dick started in the Spring of 1967 when he kindly mailed me copies of several of his pre 1968 papers, partially joint with R.P. Boas, I. Hirschman and S. Wainger, on ultraspherical expansions, transplantation theorems, mean summability and norm inequalities for some orthogonal series, etc. (see Askey and Hirschman [29], Askey and Wainger [38], and Gasper, Ismail, Koornwinder, Nevai, and Stanton [75], the Curriculum Vitae of Richard A. Askey is on pp. 19–29), and encouraged me to attend his Special Functions course, seminars, and talks during my PostDoc at the University of Wisconsin in Madison. During his talks and conversations, Dick was willing to point out many interesting open problems that he and others have been working on, encouraging others to also try to solve them. In particular, at one of his talks in February of 1968 he pointed to a complicated looking integral containing products and quotients of different sine functions that he and James Fitch needed to obtain a new shorter proof of a 1953 theorem of Turán on positivity of a certain trigonometric sum. I responded with just one word "*differentiate*" and after looking at the integral for a few seconds Dick said "*Yes, that will work!*". A few days later he handed me a preprint of the R. Askey, J. Fitch, and G. Gasper "On a positive trigonometric sum" one page paper, which was published later that year in [27]. Subsequent joint papers with Dick were a lot harder to do!

In Askey and Gasper [28], along with several other results, we used a sum of squares of Gegenbauer (ultraspherical) polynomials to prove that if $\alpha \geq -2$, then the sum of the Jacobi polynomials $P_k^{(\alpha,0)}(x)$, $k = 0, 1, ..., n$, is non-negative for $-1 < x \leq 1$, $n \geq 0$, and is equal to zero only when $\alpha = -2$ and either $n = 1$ or $x = 1$, $n \geq 1$ (now called the Askey-Gasper inequality), which was then used to prove that the Cesáro $(C, \alpha + 2)$ means of the Jacobi series of a non-negative function are non-negative when $\alpha \geq -1/2$ and $\beta = 0$. Later, sums of squares of certain Jacobi polynomials were used in Gasper [74] to prove more general inequalities and, in particular, the non-negativity of a fractional derivative of order $1/2$ of the sum in Askey-Gasper inequality, and that the Cesáro $(C, \alpha + \beta + 2)$ means of the Jacobi series of a non-negative function are nonnegative when $\alpha, \beta \geq -1/2$. The latter is best possible in the sense that all of the Cesáro (C, λ) means are not necessarily non-negative when $\lambda < \alpha + \beta + 2$. Since, due to a delay in publication, the first proofs of Askey-Gasper inequality were presented in 1975 in Askey's Regional Conference book [19] and in my survey paper [73]. Dick and I were really surprised that the Askey-Gasper inequality, which was a fractional integral of order $1/2$ of a sharper inequality, sufficed for de Branges to complete his proof of the Bieberbach conjecture (and of the more general Robertson and Milin conjectures) in [57]. Also see the papers and comments in Baernstein II, Drasin, Duren, and Marden [40].

Concerning the Askey and Wilson groundbreaking Memoir *Some basic hypergeometric orthogonal polynomials that generalize Jacobi polynomials* [39] in which they derived the orthogonality of certain balanced ${}_4F_3$ hypergeometric series and ${}_4\phi_3$ basic hypergeometric series, I wish to thank Dick for telling me that it was after seeing the Saalschützian (which he changed to the simpler word "balanced") ${}_4F_3$ series representations for the Hahn polynomials in a preprint of my paper [72, (3.18)-(3.21)] that he decided to try to discover what other orthogonal polynomials can be represented by balanced ${}_4F_3$ series and their q-analogues. It was while writing the first edition of the Gasper and Rahman *Basic Hypergeometric Series* book [76, 77], that Mizan Rahman and I decided to call the continuous ${}_4\phi_3$ orthogonal polynomials derived by Askey and Wilson in their Memoir the *Askey-Wilson polynomials.* Years later, Mizan and I derived some orthogonal multivariable generalizations of the Askey-Wilson polynomials in [78].

I also wish to thank Dick for introducing me to Special Functions, which he called *Useful Functions* in view of their applications, helping me transfer to a PostDoc position at the University of Toronto in order for my wife to satisfy a two year foreign residency requirement, recommending me for an Assistant Professor position, a Sloan Fellowship, an Associate Professor position and, later, a tenured Professor position at Northwestern University, and for many years of stimulating mathematical discussions. Most recently, last year he suggested to David C. Brown and Shaun William Davies, who in order to help make their work on Financing Efficiency of Securities-Based Crowdfunding mathematically rigorous needed a proof of a conjectured inequality for the quotient of products of certain hypergeometric series, that they contact me. For my subsequent proof and comments, see Addendum 1 in the Brown and Davies paper [55].

George Gasper
george@math.northwestern.edu

l to r: Bruce Berndt, Atul Dixit, Walter Van Assche, Dick Askey, Alan Sokal, Christian Krattenthaler, George Andrews, Patsy Wang-Iverson at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013.

Tom Koornwinder and Dennis Stanton listening to Alan Sokal at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013.

Conference table with Dick Askey at the *Askey 80th Birthday Conference*, Madison, Wisconsin, in December 2013. Clockwise: Dick Askey, Howard Cohl, Roderick Wong, Mourad Ismail, Hans Volkmer, Martin Muldoon, and Ted Chihara.

Doron Zeilberger talking at the banquet at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013.

Dick at the lectern, with Liz and family members at the family table during the banquet at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013.

Photo of persons at the Askey family table and at adjacent tables from behind the lectern before Gasper spoke during the *Askey 80th Birthday Conference* banquet, Madison, Wisconsin, USA in December. l to r: David Foss (son-in-law), Suzanne Askey (daughter), Liz Askey (wife), Jim Zurlo (son), Kathy Zurlo (daughter-in-law).

Dick Askey, Alan Sokal, George Andrews, and Peter Duren at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013.

Dick Askey and George Gasper.

Contribution #33 ——— Liber Amicorum ——— Dick Askey

From: Walter Gautschi (wgautschi@purdue.edu)
Contributor: Walter Gautschi: the conjecture of Bieberbach has become a theorem

I have had many contacts with Dick, all inspiring. The most memorable being a phone call to him in February of 1984, inquiring about certain inequalities involving integrals of Jacobi polynomials which, according to Louis de Branges, would complete his proof of the Bieberbach conjecture. As is turned out, the inequalities in question, amazingly, were special cases of inequalities established by Dick and George Gasper just a few years earlier. Thus, in no small part owing to this work of Askey and Gasper, the conjecture of Bieberbach has become a theorem. This was an event, incidentally, that helped enhancing the respect for special functions in the minds of many mathematicians, pure and applied alike.

I am also very grateful to Dick for having invited me to give a seminar on computing special functions at the University of Wisconsin during my sabbatical year of 1976/77. This provided me with a welcome incentive to take a fresh look at computational methods for special functions.

I feel very privileged to have known Dick for so many years, and I wish him well for many more to come.

Contribution #34 ——— Liber Amicorum ——— Dick Askey

From: Ralph William Gosper Jr. (billgosper@gmail.com)
Contributor: Bill Gosper: a thunderbolt from Dick prompted an extraordinary invitation

Dick Askey was huge for me. He was the first established mathematician to recognize that my amateurish series accelerations were producing new results. He connected me with George Gasper, Mizan Rahman, Mourad Ismail, Bruce Berndt, Dennis Stanton, Ira Gessel, Peter Paule, and others. He got me invited to conferences, and read with interest the results I sent to him over the years—as *Series Acceleration* became *Series Rearrangement* and finally *Path Invariant Matrix Multiplication*. Years later, he twisted the arms of the Special Functions Committee of the 2016 AMS Joint Mathematics Meetings in Seattle, Washington, Special Session on "Special Functions and q-Series" to invite my young friend Neil Bickford to speak in my place. Neil's talk was a memorable success. Yet inconceivably, MIT had recently rejected him, despite a stellar application. A thunderbolt from Dick to the Dean of Science prompted an extraordinary invitation to reverse the rejection. (How MIT botched Neil's admission remains a profound mystery to the various MIT people who knew Neil from childhood.)

It should be noted that Dick was ahead of the crowd in his enthusiasm for computer algebra. Dick liked my Macsyma results and summation algorithms, and brought others to watch my demos. I even heard him mutter to one of them: "*Hobbs class*" (meaning "First rate," as used by cricket fan G.H. Hardy. Jack Hobbs was his favorite player). At that time, the MIT Math Department was hostile to computers, driving faculty and students to switch to the EE Department. I can supply three juicy quotes:

- Prof. C.C. Lin, applied math: "*Stay away from computer—It will turn you into clerk!*"
- Famous MIT prof (Michael somebody, mentally blocked on name, lived across the street in Newton Center): "*If it is possible to pose your question to a computer, then I am not interested in it.*"
- Numerical analysis(!) grad student homework grader in response to doing in on the PDP-1 "Expensive Desk Calculator" with neatly typed out answers, vs. using pencil & paper & actual desk calculator: "*What is this shit? 0.*"

Contribution #35 ——— Liber Amicorum ——— Dick Askey

From: F. Alberto Grünbaum (albertogrunbaum@yahoo.com)
Contributor: F. Alberto Grünbaum: beyond my wildest dreams

I met Dick soon after finishing my thesis, back in 1969, when I had written a short paper on Legendre polynomials which I am sure he never liked. But we became very good friends over the years.

My best memory of him is when he introduced me at a meeting at CRM in Montreal many years later. I was talking about the bispectral problem in the discrete and continuous versions, reaching all the way to the Askey-Wilson polynomials. He started by saying "*he is not one of us, he does not work in our kind of problems, but about every ten years or so he comes up with something worth listening to.*" Dick has always had a broad view of the relations between "special functions" and the rest of math and physics. He resurrected a field that was lying semi-dormant and brought many people together into a fantastic communal enterprise, "*beyond my wildest dreams*" as he once told me. Thanks Dick!!!!

From: Mike Hirschhorn (m.hirschhorn@unsw.edu.au)
Contributor: Michael D. Hirschhorn: I had forgotten that

$$(1-x)f(0)+xf(1)-f(x)=(1-x)\int_0^x tf''(t)\,dt+x\int_x^1(1-t)f''(t)\,dt.$$

As a consequence, if $f''(t)>0$ on $(0,1)$ then the chord lies above the function.

The formula was published by itself under the title "An Identity" as a filler in the Monthly in May 1995. Dick wrote and said "*I had forgotten that.*"

An Identity

$$(1-x)g(0)+xg(1)-g(x)=$$

$$(1-x)\int_0^x tg''(t)\,dt+x\int_x^1(1-t)g''(t)\,dt$$

Submitted by Michael Hirschhorn
Department of Pure Mathematics
The University of South Wales
Kensington, New South Wales
AUSTRALIA 2033

The American Mathematical Monthly, **102**, 5 (May, 1995), p. 416.

I had discovered it while trying to prove the fact that if $f''>0$ then the chord lies above the function for my wife, who was doing an entry-level math course at University. It turned out that they only expected a hand-waving proof along the lines "If $f''>0$ then the graph is curving up like this ... , and you can see the chord is above the function."

I was thinking that Dick might like these short papers of mine, which he may not have seen.

- "A constant in a formula of Ramanujan", *The Mathematical Gazette*, **97**, 539 (2013), 276–277 [83].
- "A surprising appearance of $\zeta(3)$", *The Mathematical Gazette*, **95**, 533 (2011), 313–313 [82].
- "The AM-GM Inequality", *The Mathematical Intelligencer*, **29**, 4, (2007), 7–7 [81].

I think the first two are better than the third.
Mike Hirschhorn

Contribution #37 —— Liber Amicorum —— Dick Askey

From: Mahouton Norbert Hounkonnou (hounkonnou@yahoo.fr)
and (norbert.hounkonnou@cipma.uac.bj)
Contributor: Mahouton Norbert Hounhonnou: sincere greetings and wishes

I'd have wished to be there to celebrate with you in person, but the distance from Cotonou, in Benin, to the meeting location, the travel constraints and my administrative duties as President of Benin National Academy of Sciences, Arts and Letters, make it impossible to come.

I never met Dick in person, but his great achievements in Special Functions impacted my own life and influenced the whole African mathematical community in this field. I was introduced to Orthogonal Polynomials by Professor Andre Ronveaux from Belgium. So far, I teach and supervise African PhD students, inspired by Dick's works. In collaboration with a colleague from Poland, Professor Alina Dobrogowska, we're now working on the solutions of a general linear second order difference equation following the Askey-Wilson scheme. The resulting paper will be dedicated to Dick.

From the African community of polynomialists, I wish to express our sincere greetings and wishes, to Dick, for a happy celebration.

Contribution #38 —— Liber Amicorum —— Dick Askey

From: Plamen Iliev (iliev@math.gatech.edu)
Contributor: Plamen Iliev: beautiful blend of mathematics inspired by your work

Thank you very much, Dick, for inspiring several generations of researchers, from almost every area in pure and applied mathematics!

For me, everything started 18 years ago, when I began working on a new project with Luc Haine on bispectral extensions of the Askey-Wilson polynomials. I was immediately drawn to the beauty of the polynomials you discovered with J. Wilson and this led to my first paper on orthogonal polynomials. Once I took this road, there was no turning back! I looked at the Askey-Wilson polynomials from many different angles over the past 18 years, and every time I discovered something new and exciting.

Unfortunately, I won't be in Madison this September, but I am looking forward to the next meeting, to see the beautiful blend of mathematics inspired by your work.

Plamen

From: Mourad E. H. Ismail (mourad.eh.ismail@gmail.com)
Contributor: Mourad E. H. Ismail: the person I turn to for help, advise, or inspiration

To Dick Askey in Friendship
Mourad E. H. Ismail

It is difficult for me to write these lines knowing that Dick is not well and is in a hospice. Since 1974 when I started working with Dick, he has been the person I turn to for help, advise, or inspiration.

I first met him in 1972 when he visited Edmonton and I was a doctoral student working with Waleed Al-Salam. At the time, Waleed asked me to look at a characterization theorem that Dick suggested and it came out of a paper by Al-Salam and Chihara [3]. I could not solve the problem but I worked with Jerry Fields on another problem Dick suggested and we solved it. This resulted in my first published paper and was really a good exposure to Askey's kind of mathematics and I greatly enjoyed this work.

Both Thanaa and I attended the highly influential 1974 Conference Board of the Mathematical Sciences (CBMS) lectures at Virginia Polytechnic Institute with Dick as the principal lecturer (published in [19]). In these lectures, Dick outlined many directions for research in the field of special functions and orthogonal polynomials, and the event was an eye opener for me.

Dick kindly supported my application to spend the academic year 1974-75 at the University of Wisconsin as an assistant scientist with no teaching duties. This was an amazing year. Dick introduced me to the fascinating area of combinatorics of orthogonal polynomials. He also completely changed my outlook about what is important in Mathematics and how to do research. Through Dick, I was introduced to so many well-known mathematicians and I always appreciated this. People may not know that Dick is the reason I fell in love with Florida and its beaches. When we were working on our first combinatorics paper he suggested that I go to the combinatorics conference in Boca Raton, give a talk and ask people for references and help. The conference was 3 days long and I went to the lectures only for two days and swam in the ocean on the third day. This was in late February when Madison has a foot of snow on the ground. I did get many references on enumeration and then we did write a nice paper which appeared in the Canadian Journal of Mathematics in 1976 [31]. After we returned to Canada, I continued to go to the Boca Raton meetings and eventually we moved to Florida.

Another very memorable event happened in the summer of 1977. At the time, I was an assistant professor at McMaster University and wanted to get my brain recharged by being around Dick. I spent the whole summer in Madison. At this time, Dennis Stanton was very close to graduation, Jim Wilson had another year to go, and Dan Moak has just started. Paul Nevai was visiting the University of Wisconsin. Dick ran a seminar that met 4 times a week. Some short term visitors also attended the seminar. I remember Donald Newman and possibly others. We also had several guest speakers. George Andrews, George Gasper, and Willard Miller, Jr., also visited. There may have been others that I forgot. Many papers resulted from this seminar. Paul Nevai and I presented the Pollaczek memoir [103] and I started working with Dick on what evolved into our 1984 AMS memoir [30]. We also started the continuous q-ultraspherical polynomials paper [33]. I have never slept as little as I did during that summer.

Over the years, I continued my collaboration with Dick and I learned a lot of mathematics just from talking to him. Dick also wrote numerous letters of recommendation for me when I was applying for jobs. He also evaluated my work for promotion and tenure. Dick was always very generous with his time and ideas. He solved many important problems in special functions and orthogonal polynomials and posed many more. He attracted many good young people to the subject. He also helped many of us in our early careers to publish our papers and advance in our careers.

When the Askey-Wilson polynomials were found, Dick called them the q-Wilson polynomials. Based on many hints from Dick, Wilson identified the Wilson polynomials. I would like to claim credit for disagreeing with Dick about naming the polynomials "q-Wilson polynomials" and when I discussed the matter with Mizan Rahman and Dennis Stanton, they agreed with me. So the three of us engineered a mutiny and insisted on using "The Askey-Wilson polynomials". Eventually Dick relented. Once, I was discussing certain issues with Persi Diaconis. In the middle of the discussion he said that a hundred years from now the Askey-Wilson polynomials will be still around.

Thank you Dick for all you did for me personally and for all the mathematicians of my generation. Most importantly, thank you for making me and Thanaa part of your family.

Mourad

Dick Askey and Mourad Ismail at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013. Photo taken by Patsy Wang-Iverson.

Dennis Stanton, Dick Askey and Mourad Ismail
at Madison, Wisconsin, USA in September 2019.

From: Tatsuro Ito (tito@staff.kanazawa-u.ac.jp)
Contributor: Tatsuro Ito: Askey-Wilson polynomials in relation to quantum affine groups

Dear Prof. Askey,

This is Tatsuro Ito. I have been collaborating with Paul Terwilliger since he introduced the concept of Leonard pairs in the mid-eighties, just after Eiichi Bannai and I published the book "Algebraic Combinatorics I: Association Schemes".

I visited Ohio State University from 1980-81. It was the time when Eiichi proposed the study of (P and Q)-polynomial association schemes in the series of his lectures there and Doug Leonard found that their eigenmatrices are expressed in terms of the Askey-Wilson polynomials. I am now in Anhui University, Hefei, China, having turned the retirement age 65 in Kanazawa University, Japan, 5 years ago.

Around the year I moved to China, I wrote an article in Japanese on the tridiagonal pairs and the q-Onsager algebra, in which I explained how Paul and I developed the theory of Askey-Wilson polynomials in relation to quantum affine groups. It is now translated into English and is going to be published in the AMS Sugaku Expositions, perhaps in a few months.

The case of $q = +1, -1$ is not yet completed, because we do not have the quantum affine groups defined properly at $q = +1, -1$. We plan to celebrate Paul's 65th birthday next summer in Slovenia, and I hope some progress could be reported on that occasion.

Sincerely, Tatsuro Ito

From: Warren Johnson (wpjoh@conncoll.edu)
Contributor: Warren P. Johnson: trying to educate the entire mathematical community

I was a directionless student in high school until falling in love with techniques of integration late in my senior year. While there were occasional highlights, such as Laplace transforms and residue calculus, and I did slowly learn to like some other parts of mathematics, my undergraduate years weren't much better. My first year of graduate school was so unpleasant that I was just about ready to abandon the idea of becoming a mathematician. When I took Dick's Special Functions course in my third semester, my life changed.

It was the first time I found someone who loved integrals as I did, and I saw immediately that Dick knew far more about them than anyone I'd ever met. On the first day he connected the beta integral to the gamma function using functional equations, a breathtaking argument that I have since used several times in my own classes. As a final project, he had me work through his paper "*More q-beta integrals*" (joint with Ranjan Roy) [35]. This taught me what it really meant to read a significant piece of mathematics, and the realization that I could actually do it was a tremendous thrill. In this context, I must also mention "*An Elementary Evaluation of a Beta Type Integral*" [20], one of my favorite papers. I think Dick was not entirely satisfied with the original evaluation of the Askey–Wilson integral, in spite of its importance. Here he evaluates a generalization by a *tour de force* of functional equations.

Dick also opened up the world of q to me, which became my second great love in mathematics. In his honor, I want to say a little about an elementary idea in integration by parts. Examples are scattered in the literature, but it seems that no one has ever written it up systematically. When I showed it to Dick some years ago, he told me he had not seen it before. That will surprise anyone who knows him.

I'll start with an integral that appears in three great old calculus books, with three different treatments. (Reading great old mathematics is another of Dick's gifts to me.) We can write

$$\begin{aligned}\int \frac{xe^x}{(x+1)^2}\,dx &= \int \frac{(x+1)e^x - e^x}{(x+1)^2}\,dx \\ &= \int \frac{e^x}{x+1}\,dx - \int \frac{e^x}{(x+1)^2}\,dx.\end{aligned} \tag{E}$$

If you know a lot about about integrals, these both look hopeless. If you only know a little, it is natural to integrate one of them by parts, say $u = e^x$ and $dv = dx/(x+1)^2$, so $du = e^x\,dx$ and $v = -1/(x+1)$. Then

$$\int \frac{xe^x}{(x+1)^2}\,dx = \int \frac{e^x}{x+1}\,dx - \left(\frac{-e^x}{x+1} + \int \frac{e^x}{x+1}\,dx\right) = \frac{e^x}{x+1} + C.$$

This is essentially Lacroix's solution [87, p. 94]. I like to describe it by saying that the integral *commits suicide*. Euler [70, p. 145, Section 233] more or less recognizes (E) as a quotient rule, and Bertrand [49, p. 10] integrates by parts at the beginning with $u = xe^x$ and $dv = dx/(x+1)^2$.

Many of the suicidal integrals I have seen are based on the fact that

$$\frac{d}{dx}\,\frac{\sin x}{1+\cos x} = \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \frac{1}{1+\cos x},$$

and consequently

$$\int \frac{f'(x)\sin x + f(x)}{1+\cos x}\,dx = \frac{f(x)\sin x}{1+\cos x} + C.$$

The examples $f(x) = e^x$ and $f(x) = x$ are in several places in the literature. Edwards gives one or two others in his *Treatise on the Integral Calculus* [63, 64].

This is not the place to multiply examples. I'll conclude with one that I was constructing at the moment the power went out when Tropical Storm Irene hit the Connecticut shoreline in 2011. Rewrite

$$\int e^{x\sqrt{2}}\tan^3 x\,dx = \int e^{x\sqrt{2}}\tan x\sec^2 x\,dx - \int e^{x\sqrt{2}}\tan x\,dx,$$

and do *both* integrals by parts. In the first take $u = e^{x\sqrt{2}}$ and $dv = \tan x\sec^2 x\,dx$, so that $v = \frac{1}{2}\sec^2 x$ and $du = \sqrt{2}\,e^{x\sqrt{2}}\,dx$. In the second take $u = \tan x$ and $dv = e^{x\sqrt{2}}\,dx$, so that $du = \sec^2 x\,dx$ and $v = \frac{1}{\sqrt{2}}\,e^{x\sqrt{2}}$. Then we have

$$\begin{aligned}\int e^{x\sqrt{2}}\tan^3 x\,dx &= \frac{1}{2}\,e^{x\sqrt{2}}\sec^2 x - \frac{1}{\sqrt{2}}\int e^{x\sqrt{2}}\sec^2 x\,dx \\ &\qquad - \left(\frac{1}{\sqrt{2}}\,e^{x\sqrt{2}}\tan x - \frac{1}{\sqrt{2}}\int e^{x\sqrt{2}}\sec^2 x\,dx\right) \\ &= \frac{1}{2}\,e^{x\sqrt{2}}\sec^2 x - \frac{1}{\sqrt{2}}\,e^{x\sqrt{2}}\tan x + C.\end{aligned}$$

This is the simplest of a family of integrals. The next two members are

$$\int e^{x\sqrt{2}}\tan^2 x\sec^2 x\,dx \qquad \text{and} \qquad \int e^{x\sqrt{2}}\left(2\tan^5 x + \tan x\right)dx.$$

At least from my perspective, the late 1980s were not a great time for special functions at Wisconsin. There were strong postdocs (first Frank Garvan and then Peter Forrester), but interest among the other graduate students was minimal. Fortunately, I couldn't have cared less what they thought—if I hadn't learned all the mathematics I ought to have done by then, at least I had learned that. Things picked up in the early 1990s after Shaun Cooper came. But what made Dick unique is that he wasn't just trying to educate the students at Wisconsin, he was trying—and in a large measure succeeding—to educate the entire mathematical community. (Even this is not broad enough, since he often talked to people in other disciplines, especially physics.) He is irreplaceable.

From: Diana Kasbaum (diana.kasbaum@gmail.com)
Contributor: Diana Kasbaum: Dick knew the locations of every book!

I first came to know Richard Askey as a first semester freshman at the University of Wisconsin-Madison in 1970. Dick was professor for the Calculus 222 course—the same one where I met my future husband. My affiliation with Dick resumed again in 2001 when I left my teaching and elementary mathematics coordinator position and began a career as a mathematics consultant for the Wisconsin Department of Public Instruction. I was a co-presenter on session focusing on NAEP (National Assessment of Educational Progress). In true Dick Askey fashion, he was frustrated with aspects of NAEP and shared his concerns with the WI NAEP coordinator. This meeting gave me the opportunity to reintroduce myself to Professor Askey, letting him know that I met my husband in his Calc 222 class in the fall of 1970. Immediately, I saw the softer side of Professor Askey. From that time forward, Dick would contact me regularly with his concerns regarding the teaching and learning of mathematics, especially at the elementary level. While we didn't always agree, he caused me to think deeply about the direction of mathematics standards, curriculum and instruction. He was always a proponent of Singapore math and was very willing to share resources. I recall going to his home on Regent Street in Madison, Wisconsin, not far from the University where Dick and his wife, Liz, welcomed me. I was then treated to a trip into the basement of his turn of the century home where it was like stepping into the archives of mathematics history. There were rows of floor-to-ceiling shelves where it seemed that every mathematics book ever published was neatly housed AND Dick knew the location of every book!

Dick was a regular presenter at the Wisconsin Mathematics Council's Annual Conference in Green Lake, sharing insights, K-12. We served on the initial committee of the Illustrative Mathematics Project and were advisors for the Mathematics Institute of Wisconsin (formerly the Brookhill Foundation). We were often on the same flights to meetings where Dick invited me to be his guest in the Delta Sky Miles Club where he challenged me with complex math problems while waiting for a connecting flight at the Minneapolis or Detroit airports. He was very patient as I tried to resurrect the higher level math hidden somewhere in my aging brain.

What I remember most about Professor Askey is his kindness during two important events in my life. In September 2014, Dick came to my mother's funeral visitation. I was so surprised that he took the time to pay his respects, not even knowing my mom. I was delighted to reintroduce Dick to my husband of 40 years. Again in October 2015, I was honored when Dick came to my retirement gathering at the WI Department of Public Instruction. While he had many contributions to national and international mathematics, what I will remember most is the softer, kinder side of Professor Askey.

Diana Kasbaum
Retired, WI Department of Public Instruction
Association of State Supervisors of Mathematics, President 2011-14
Wisconsin Mathematics Council (WMC), President 2009-11
WMC Distinguished Mathematics Educator Award, 2006
University of Wisconsin-Madison Distinguished Elementary Education Alumni Award, 2001
Presidential Award for Excellence in Mathematics and Science Teaching, 1999

From: Anatol Kirillov (kirillov@kurims.kyoto-u.ac.jp)
Contributor: Anatol Kirillov: a great Mathematician and Person

Dick Askey, an outstanding Mathematician and Great Person who supervised and helped many of us to find our own place in the Life and Science. Dick, a wonderful teacher, made considerable contribution to many fields of Mathematics, including Special Functions, Combinatorics, Integrable Systems among many others. His discovery of the Askey-Wilson polynomials made a revolution in the Theory of Orthogonal Polynomials! A great Mathematician and Person.

From: Tom H. Koornwinder (T.H.Koornwinder@uva.nl)
Contributor: Tom H. Koornwinder: germs for multi-variable analogues

Formulas for one-variable special functions as germs for multi-variable analogues
Tom H. Koornwinder

Subsection 1: Introduction

Dick Askey spent the academic year 1969–1970 at the *Mathematisch Centrum* in Amsterdam, where I had started working as a PhD student a little earlier. See the slides [84] of my lecture at Askey's 80th and Nico Temme's contribution to the present Liber Amicorum for an account of that year and the impact of Dick's stay. For me it meant that I specialized in orthogonal polynomials and special functions (OPSF) with group theoretic interpretation, and that I spent the next academic year at the Mittag-Leffler Institute in Djursholm, Sweden, where the whole year was devoted to noncommutative harmonic analysis. Moreover I took Dick's open problem about the addition formula for Jacobi polynomials with me to Sweden, and I solved it there.

In the fifty years since I met Dick first I heard many lectures by him. Quite often he mentioned addition formulas, maybe only in the simple case of Legendre polynomials $P_n(x)$, and he always added that there should be a dual addition formula, which leaves x fixed rather than n, and manipulates n rather than x. I always found this a strange idea which I did not take very seriously, until, in 2016, I suddenly got a brain wave when looking at a formula in a preprint by Hallnäs and Ruijsenaars and could produce the dual addition formula for ultraspherical polynomials [85]. Dick was happy when I wrote this to him.

Dick always saw the importance of extending OPSF to several variables, although he worked himself usually only in one variable. He meant of course extensions which have sufficient depth, for instance by a Lie theoretic connection, or because the more variable functions admit analogues of the famous evaluation and transformation formulas for ${}_rF_s$ and ${}_r\phi_s$ (q-)hypergeometric functions. Usually, the formulas in several variables are much more complicated than their one-variable analogues. Still a good knowledge of the one-variable case can be helpful to predict certain phenomena in the multi-variable case. I think I learnt this approach from Dick. This was for instance successful when I could obtain in 1987 the Pieri formula for (A-type) Macdonald polynomials by being aware of the duality for continuous q-ultraspherical polynomials which is obvious from one of its q-hypergeometric representations.

In this note I will present one-variable cases of A-type and BC-type interpolation polynomials, and I will discuss some perspectives for the several variable cases in their interaction with the one-variable cases.

Acknowledgment: I thank Michael Schlosser and Ole Warnaar for helpful remarks.

Subsection 2: Interpolation polynomials

(q-)Pochhammer symbols

Let $0<|q|<1$, $n\in\mathbb{Z}_{\ge 0}$,

$$\begin{aligned}(a)_n &:= a(a+1)\dots(a+n-1),\\ (a;q)_n &:= (1-a)(1-qa)\dots(1-q^{n-1}a),\\ (a_1,\dots,a_k;q)_n &:= (a_1;q)_n\dots(a_k;q)_n.\end{aligned}$$

These omnipresent building blocks of one-variable special functions are known to everybody. Less known or less used is that they can be characterized by an interpolation property:

1. $z(z-1)\dots(z-n+1)=(-1)^n(-z)_n$
   is the unique monic polynomial of degree n which vanishes at $z=0,1,\dots,n-1$.
2. $z(z-q)\dots(z-q^{n-1})=z^n(z^{-1};q)_n$
   is the unique monic polynomial of degree n which vanishes at $z=1,q,\dots,q^{n-1}$.
3. $\displaystyle\prod_{j=0}^{n-1}(z+z^{-1}-aq^j-a^{-1}q^{-j})=\frac{(az,az^{-1};q)_n}{(-1)^n\,q^{\frac12 n(n-1)}\,a^n}$
   is the unique monic symmetric Laurent polynomial of degree n which vanishes at $z=a,aq,\dots,aq^{n-1}$ (and their inverses).

Subsection 3: Generalized binomial formulas

You may be surprised to see the following formula being called a binomial formula:

$$\begin{aligned}R_n(z;a,b,c,d\,|\,q)&:=\frac{p_n(\frac12(z+z^{-1});a,b,c,d\,|\,q)}{p_n(\frac12(a+a^{-1});a,b,c,d\,|\,q)}={}_4\phi_3\left(\begin{matrix}q^{-n},q^{n-1}abcd,az,az^{-1}\\ ab,ac,ad\end{matrix};q,q\right)\\ &=\sum_{k=0}^n\frac{q^k}{(ab,ac,ad,q;q)_k}\,(q^{-n},q^{n-1}abcd;q)_k\,(az,az^{-1};q)_k.\end{aligned}\tag{4}$$

It is the well-known q-hypergeometric representation of suitably normalized Askey-Wilson polynomials. However, with z replaced by az and $a\to\infty$ it yields a q-binomial formula

$$\begin{aligned}z^n={}_2\phi_0\left(\begin{matrix}q^{-n},z^{-1}\\ -\end{matrix};q,q^nz\right)&=\sum_{k=0}^n\frac{(-1)^kq^{-\frac12k(k-1)}}{(q;q)_k}\,q^{nk}(q^{-n};q)_k\,z^k(z^{-1};q)_k\\ &=\sum_{k=0}^n\frac{(q;q)_n}{(q;q)_k(q;q)_{n-k}}\,(z-1)(z-q)\dots(z-q^{k-1}).\end{aligned}\tag{5}$$

The first equality sign in (5) follows from a confluent case of the q-Chu-Vandermonde formula [77, (II.7)]. It is less evident to see in a straightforward way that $R_n(az;a,b,c,d\,|\,q)\to z^n$ as $a\to\infty$. However, by the symmetry of the Askey-Wilson polynomials in a,b,c,d we have

$$R_n(az;a,b,c,d\,|\,q)=\frac{a^n(bc,bd;q)_n}{b^n(ac,ad;q)_n}\,{}_4\phi_3\left(\begin{matrix}q^{-n},abcdq^{n-1},abz,a^{-1}bz^{-1}\\ ab,bc,bd\end{matrix};q,q\right),$$

from which the limit is clear. From (5) we obtain in the limit for $q\to1$ the classical binomial formula

$$z^n={}_1F_0\left(\begin{matrix}-n\\ -\end{matrix};z-1\right)=\sum_{k=0}^n\binom{n}{k}(z-1)^k.\tag{6}$$

Formulas (6), (5) and (4) have d-variable analogues which are called binomial formulas for Jack polynomials [102], (A-type) Macdonald polynomials [100] and Koornwinder polynomials [101], respectively. Formulas (4) and (5) involve an expansion of the left-hand side in terms of interpolation polynomials in z, respectively given by item 3 and item 2 above. Moreover, they are as well expansions in terms of interpolation polynomials depending on q^n. This is clear for (5), while in (4) we can rewrite

$$(q^{-n},q^{n-1}abcd;q)_k=\big(\,\tilde a\,(\tilde a q^n),\tilde a\,(\tilde a q^n)^{-1};q\big)_k,\quad\text{where}\quad \tilde a:=(q^{-1}abcd)^{\frac12}.$$

This also implies dualities. In (5) put $z=q^m$. Then

$$q^{mn}={}_2\phi_0\left(\begin{matrix}q^{-n},q^{-m}\\ -\end{matrix};q,q^{m+n}\right)=\sum_{k=0}^{\min(m,n)}\frac{(-1)^k q^{-\frac12 k(k-1)}}{(q;q)_k}\,q^{nk}(q^{-n};q)_k\,q^{mk}(q^{-m};q)_k$$

with evident $m\leftrightarrow n$ symmetry in all parts.

For (4) introduce dual parameters $\tilde a,\tilde b,\tilde c,\tilde d$:

$$\tilde a:=(q^{-1}abcd)^{\frac12},\quad \tilde a\,\tilde b=ab,\quad \tilde a\,\tilde c=ac,\quad \tilde a\,\tilde d=ad.$$

Then we have the duality relation

$$R_n(a^{-1}q^{-m};a,b,c,d\,|\,q)=R_m(\,\tilde a^{-1}q^{-n};\tilde a,\tilde b,\tilde c,\tilde d\,|\,q)\quad(m,n\in\mathbb{Z}_{\ge0}),$$

since both sides are equal to

$$\sum_{k=0}^{\min(m,n)}\frac{q^k}{(ab,ac,ad,q;q)_k}\,(q^{-n},\tilde a^2q^n;q)_k\,(q^{-m},a^2q^m;q)_k.$$

All this generalizes to the d-variable case of Macdonald polynomials [100] and Koornwinder polynomials [101]. The d-variable analogue [101] of (4) is, in a sense, an explicit expression for the Koornwinder polynomials.

Subsection 4: Further one-variable binomial formulas

Above the q-binomial formula (5) lies a version of the q-Chu-Vandermonde formula [77, (II.7)]

$$\frac{(az;q)_n}{(a;q)_n}={}_2\phi_1\left(\begin{matrix}q^{-n},z^{-1}\\ a\end{matrix};q^naz\right)=\sum_{k=0}^n\frac{a^k}{(a,q;q)_k}\,q^{nk}(q^{-n};q)_k\,z^k(z^{-1};q)_k. \tag{7}$$

Indeed, (7) yields (5) in the limit for $a\to\infty$. Formula (7) can be considered as a binomial formula for the interpolation polynomials in item 2 in Subsection 2. Both in the variable z and the variable q^n it expands in terms of interpolation polynomials. For $z=q^m$ we have again the $m\leftrightarrow n$ symmetry in all parts of the formula.

Above (7) lies a version of the q-Saalschütz sum [77, (II.12)]

$$\frac{(ac^{-1}z,ac^{-1}z^{-1};q)_n}{(a,ac^{-2};q)_n}={}_3\phi_2\left(\begin{matrix}q^{-n},cz,cz^{-1}\\ a,c^2a^{-1}q^{1-n}\end{matrix};q,q\right)=\sum_{k=0}^n\frac{(q^{-n};q)_kq^k}{(a,c^2a^{-1}q^{1-n},q;q)_k}\,(cz,cz^{-1};q)_k. \tag{8}$$

Indeed, in (8) replace z by cz and let $c\to\infty$. Then we obtain (7). Formula (8) can be considered as a connection formula between interpolation polynomials for two different parameters in item 3 in Subsection 2. It is tempting to call (8) a binomial formula for these interpolation polynomials.

Quite surprising for me (not having known this before I wrote this note), formula (8), in its turn, is a limit case of (4). In order to see this we combine (4) with the symmetry of the Askey–Wilson polynomial p_n in the parameters and use the evaluation

$$p_n(\tfrac12(a+a^{-1});a,b,c,d\,|\,q)=a^{-n}(ab,ac,ad;q)_n$$

to obtain

$$\begin{aligned}
&p_n(\tfrac12(z+z^{-1});a,b,c,d\,|\,q)\\
&=a^{-n}\sum_{k=0}^n\frac{q^k}{(q;q)_k}\,(abq^k,acq^k,adq^k;q)_{n-k}\,(q^{-n},q^{n-1}abcd;q)_k\,(az,az^{-1};q)_k\\
&=b^{-n}\sum_{k=0}^n\frac{q^k}{(q;q)_k}\,(abq^k,bcq^k,bdq^k;q)_{n-k}\,(q^{-n},q^{n-1}abcd;q)_k\,(bz,bz^{-1};q)_k.
\end{aligned}$$

Now put $c=a^{-1}q^{-n+1}$. Then

$$\begin{aligned}
&p_n(\tfrac12(z+z^{-1});a,b,a^{-1}q^{-n+1},d\,|\,q)=\frac{q^n(q^{-n},bd;q)_n}{a^n(q;q)_n}\,(az,az^{-1};q)_n\\
&=b^{-n}(bd;q)_n\sum_{k=0}^n\frac{q^k}{(q;q)_k}\,(abq^k,a^{-1}bq^{-n+k+1};q)_{n-k}\,(q^{-n},q^{n-1}abcd;q)_k\,(bz,bz^{-1};q)_k.
\end{aligned}$$

Replace a,b by ac^{-1},c. After a few manipulations we recover (8). In particular, d is no longer present in the identity. We see also that the interpolation polynomials in item 3 of Subsection 2 are specializations of Askey–Wilson polynomials. I got the idea from a similar specialization to interpolation functions of Spiridonov's multivariable biorthogonal elliptic functions, a formula due to Rains [105] and given more explicitly in [111, (1.4.21)] (there replace μ by λ on the right).

So we have a chain of limits of functions

$$p_n(z;a,b,c,d\,|\,q)\;\to\;(az,az^{-1};q)_n\;\to\;(z;q)_n\;\to\;z^n,$$

where the second and the third are interpolation functions. We have a corresponding chain of limits of "binomial formulas" (4) $\to$ (8) $\to$ (7) $\to$ (5).

Subsection 5: A few comments on analogues in several variables

I conclude this note with a few somewhat vague thoughts (to be made more concrete if a little more time would be given). For these I climb up to the realm of elliptic hypergeometric series, a nowadays very active area which has also been enthusiastically welcomed by Dick.

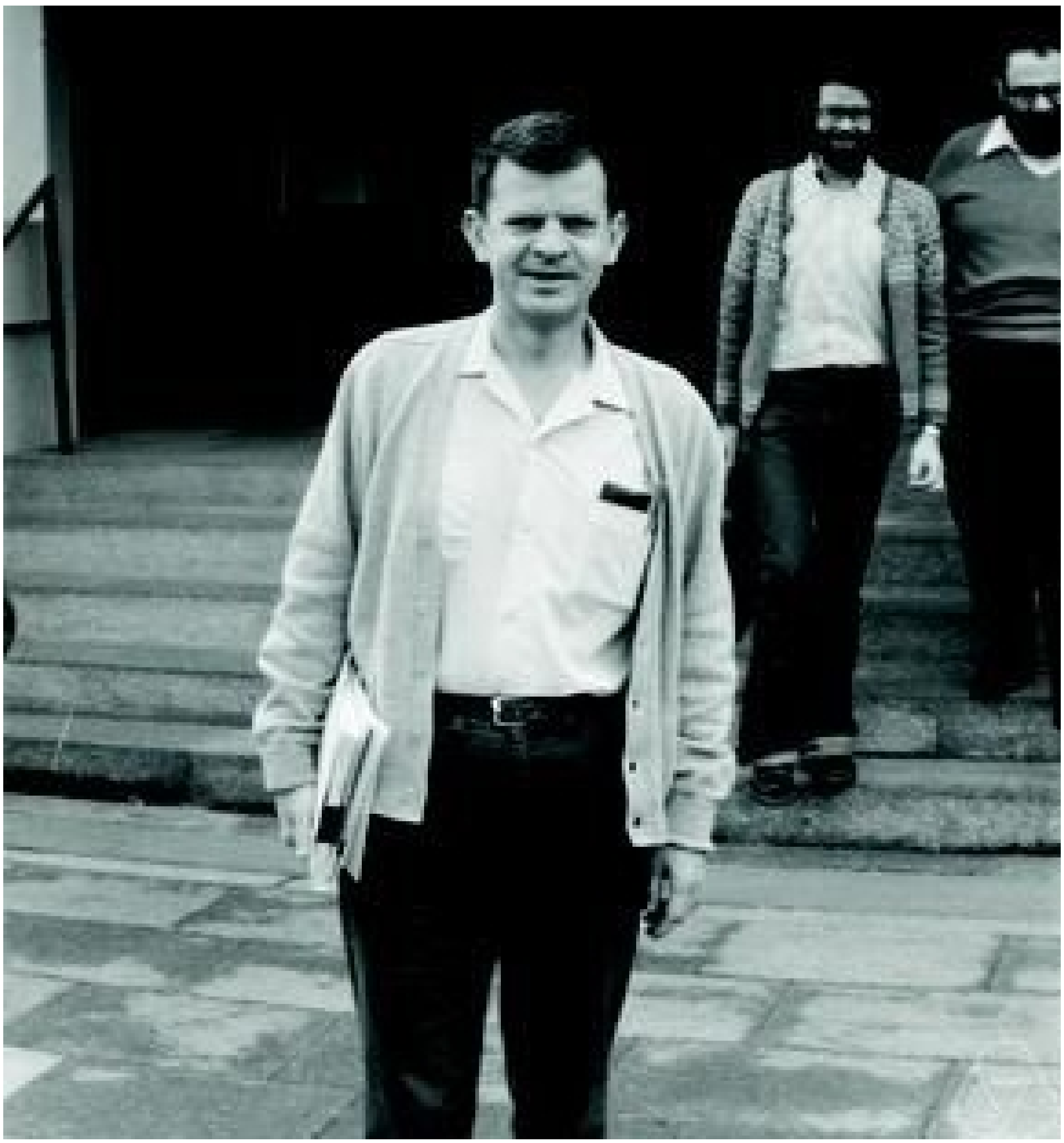
1. I already mentioned the specialization of Askey–Wilson polynomials to the interpolation polynomials $(az,az^{-1};q)_n$, which is given on a much higher level in [111, (1.4.21)]. Certainly, a suitable limit for $p\to0$ of this last formula will give a specialization of Koornwinder polynomials to Okounkov's interpolation polynomials [101]. It is quite remarkable that the A-type interpolation polynomials (see for instance [100]) generalize the A-type Macdonald polynomials, while the converse is true in the BC-case.
2. Note the elliptic analogue [77, (11.4.11)] of the q-Saalschütz sum. Just as for (8) this can be rewritten as a connection formula for elliptic interpolation functions, and such that it has (8) as a limit case for $p\to0$. See Schlosser [114, Example 4.3]. There the elliptic interpolation functions are

$$\frac{(bz,bz^{-1};q,p)_n}{(cz,cz^{-1};q,p)_n},$$

in terms of elliptic analogues of Pochhammer symbols. In fact, [77, (11.4.11)] is a rewritten form of the Frenkel–Turaev sum ${}_{10}V_9$ sum [77, (11.4.1)]. A limit on a higher level for $p \to 0$ of this sum is to Jackson's ${}_8\phi_7$ sum [77, (2.6.2)]. Accordingly the elliptic interpolation functions tend to the rational interpolation functions occurring in the expansion of Rahman's biorthogonal rational functions.

3. Rosengren & Warnaar [111, (1.4.19)] give a connection formula for d–variable elliptic interpolation polynomials, which is due to Rains [105, Corollary 4.14]. Limits for $p \to 0$ of this formula, finally leading to (8) in the one-variable case, can certainly be given by using [105, Section 8] and [104].

Dick, my best wishes for you. If you are in the mood, look at the math in this note. If you are not in the mood, then just delight in knowing that you have stimulated so many young, and sometimes older, people in their mathematical development.



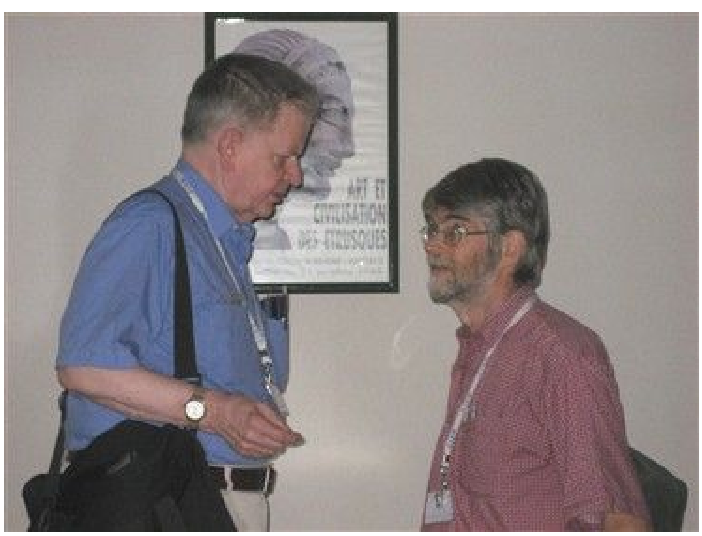
Dick Askey at Oberwolfach, Germany

Dick Askey at The Netherlands in 1970

Dick Askey in Evanston, Illinois in 1980

Dick Askey in Evanston, Illinois in 1983

Dick Askey and Tom Koornwinder at Dick Askey's *80^{th} Birthday Conference* in Madison, Wisconsin, USA. Photo taken by Patsy Wang-Iverson.



Dick Askey and Tom Koornwinder at Luminy in July 2007.

From: Christian Krattenthaler (christian.krattenthaler@univie.ac.at)
Contributor: Christian Krattenthaler: Richard has **always** been like that, already as a young man!

Obviously, I knew the name "Richard Askey" already as a student, through his work on orthogonal polynomials and q-series. I remember in particular studying the great Memoirs volume of his and Mourad Ismail [30] which made a big (and lasting) impression on me.

When I met Richard in person I do not remember for sure; it may have been at an AMS meeting in 1994 or at a Fields Institute Workshop in 1995. It goes without saying that I was struck by his personality. On the other hand, I can't report any specific encounters that made a direct impact on me or on my career (that may have been different behind the scenes, but that I do not know); although he was always very encouraging, our research interests were not close enough I guess.

Instead I want to quote Dominique Foata on Richard Askey. Dominique once told me: "*If, in a referee's report, you read somewhere "...and this is very important!", then this is a report by Richard Askey.*" (I don't know whether this is true; if so, I never got a report from Richard Askey ...) We all know Richard Askey as somebody who acts as a preacher and prophet, telling people which research problems they should look at, which methods may work, which research directions look promising, etc., frequently against the "main stream". Dominique added that Richard has **always** been like that, already as a young man! This is very impressive since this requires a lot of courage, but most importantly a most profound knowledge and taste. As an example, Dominique mentioned his bijective proof of the Mehler formula for Hermite polynomials. Dominique told me that he himself did not think greatly of it, he considered it a nice exercise, but nobody would be interested in it. Therefore, he did not even want to publish it. Richard insisted that he **had** to publish it. As we all know today, the consequence was a whole flood of research by so many people (continuing up to today) showing that there is much more combinatorics in analysis and leading to so much more insight than had existed before, and it made Dominique an invited speaker at the ICM in Warszawa in 1983.

No doubt, Richard Askey could also be very critical. As it fits to a personality of his caliber, this criticism did not stop in front of him. I have heard Richard tell the following at least twice in talks that he gave, but—unfortunately—it does not seem to be written down in one of his articles. Fortunately, Doron Zeilberger recorded it in his article that contains his proof of the refined alternating sign matrix conjecture [134, p. 63]. The story begins with Richard's firm belief that the determinantal formula for orthogonal polynomials in terms of the moments of the orthogonality measure is aesthetically pleasing but otherwise completely useless. This belief was destroyed by his student Jim Wilson who used exactly that formula to find the—what are now called—Wilson polynomials, and subsequently led to the discovery of the—what are now called—Askey–Wilson polynomials. After having told this, Richard then gave the following advice to young people: "*If an authority in the field tells you that you should look at a certain thing, listen! If that authority tells you to* **not** *look at a certain thing,* **don't** *listen!*"

This is wisdom at its purest. Richard Askey stands and stood exactly for this.

From: Zhi-Guo Liu (liuzg@hotmail.com)
Contributor: Zhi-Guo Liu: professor Askey's influence on me in mathematics

In the late 1980s, inspired by the paper "The Quarterly Reports of S. Ramanujan, The American Mathematical Monthly 1983(90) 505-516." due to Bruce Berndt [43], I began to study Ramanujan's mathematics in isolation in China. In the early 1990s, I began to study q-series and theta functions.

The four most influential papers for me in q-series are (1) George E. Andrews, "The fifth and seventh order mock theta functions" [10]; (2) Richard Askey and James Wilson, "Some basic hypergeometric orthogonal polynomials that generalize Jacobi polynomials" [39]; (3) S. Roman, "More on the umbral calculus, with Emphasis on the q-umbral calculus" [110]; (4) Richard Askey and M. E. H. Ismail, "The very well-poised ${}_6\psi_6$" [32].

Because I lacked knowledge of orthogonal polynomials at that time, Askey-Wilson's polynomials was the most difficult for me. I was conquered by the beauty and profundity of Askey-Wilson's polynomials and it took me about 20 years to understand these polynomials in my own way. At around 2010, I found a simple generating function for the Askey-Wilson polynomials which allowed me to give a new proof of the orthogonality relation for the Askey-Wilson polynomials.

During the *Mysore Conference to Commemorate the 125th Anniversary of Ramanujan's Birth* in 2012, I showed my result to Professor Askey. He liked and encouraged me to prove the orthogonality relation for the big q-Jacobi polynomials. These two research results have been published in the Ramanujan Journal in 2013 [89] and the Journal of Mathematical Analysis and Their Applications in 2014 [90], respectively. In my recent paper in the Proceedings of the American Mathematical Society, I have used the orthogonality relation for the Askey-Wilson polynomials to a double q-series transformation formula with twelve parameters [91].

In the summer of 2013, Professor Askey visited East China Normal University. I still remember clearly his visit. He participated in the q-series workshop of East China Normal University and he made a brilliant report entitled as "Some positivity results for sums of Jacobi polynomials."

In order to promote the development of mathematics in East China Normal University, the University awarded Professor Askey the title of honorary professor in 2014. Unfortunately, for health reasons, he was unable to visit East China Normal University again.

During the number theory conference in honor of Krishna Alladi's 60th birthday, I met Professor Askey again and we had a pleasant conversation at Professor Li-Chien Shen's house. He encouraged me to continue my research on q-partial differential equations.

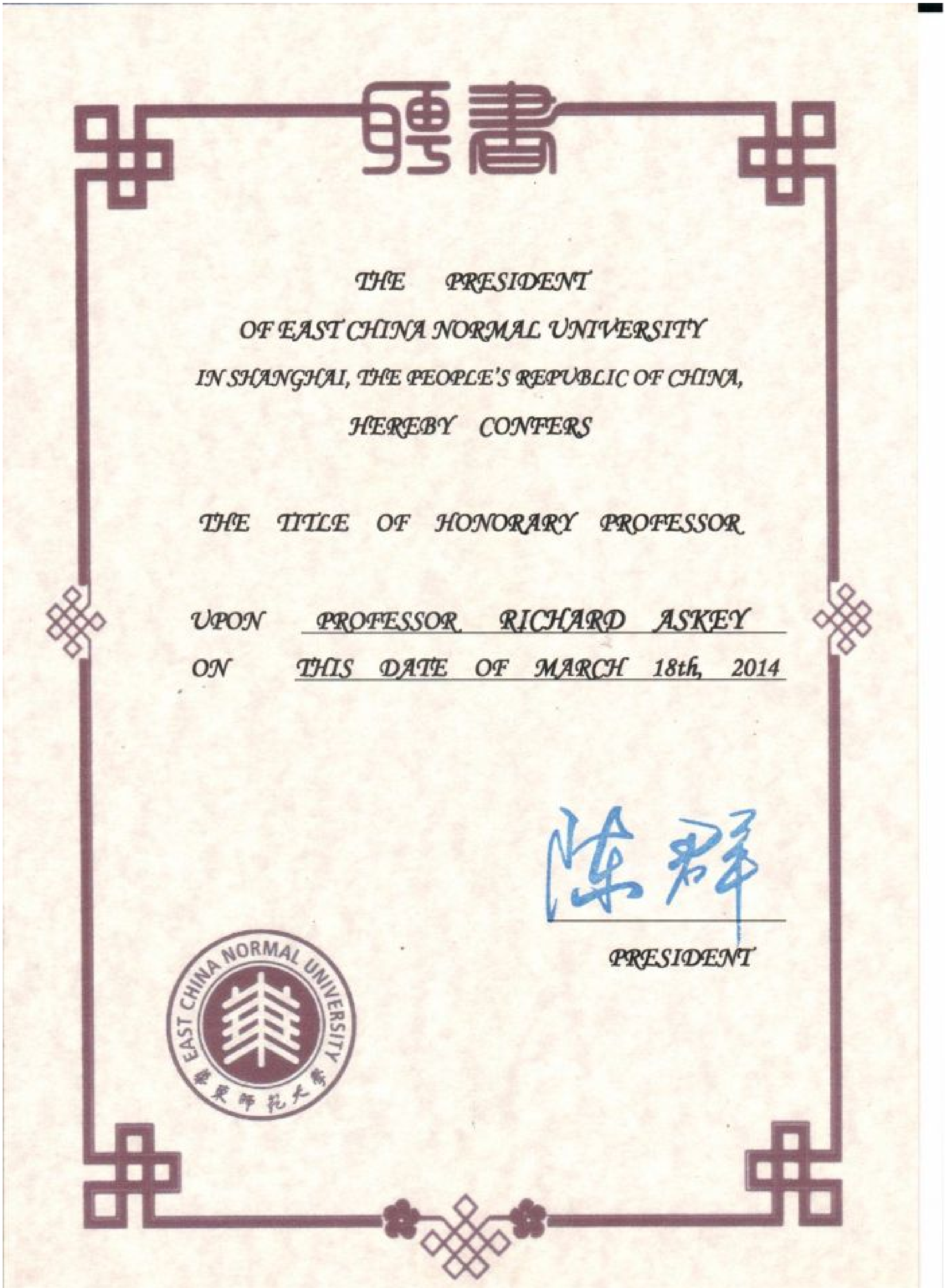
I've met Professor Askey five times in total and the first time I met him was at the conference *q-Series with Applications to Combinatorics, Number Theory, and Physics* held in the University of Illinois at Urbana-Champaign in 2000.

In short, Professor Askey is a mathematician who has a great influence on me in mathematics. I sincerely hope that he can recover soon and continue his research in special functions and q-series.

Zhi-Guo Liu
East China Normal University, Shanghai, China

Photo taken during "q-Series with Applications to Combinatorics, Number Theory and Physics," October 26-28, 2000, University of Illinois, Urbana-Champaign, Illinois. l to r: Ken Ono, Bruce Berndt, Sander Zwegers, Dick Askey, Zhi-Guo Liu, Neville Robbins. Ken Ono indicated that this was the day that Zwegers announced his work on mock theta functions.

The photo was taken during the conference *The Legacy of Srinivasa Ramanujan* held in 2012 at the University of Delhi. l to r: Dick Askey, George Andrews, Bruce Berndt, Zhi-Guo Liu.



聘書

THE PRESIDENT
OF EAST CHINA NORMAL UNIVERSITY
IN SHANGHAI, THE PEOPLE'S REPUBLIC OF CHINA,
HEREBY CONFERS

THE TITLE OF HONORARY PROFESSOR

UPON PROFESSOR RICHARD ASKEY
ON THIS DATE OF MARCH 18th, 2014

陈群
PRESIDENT

EAST CHINA NORMAL UNIVERSITY
華東師範大學

The scanned copy of Professor Askey's Honorary Professor's Certificate.

From: Lisa Lorentzen (lisa.lorentzen@ntnu.no) and Frode Rønning (frode.ronning@ntnu.no)
Contributor: Lisa Lorentzen and Frode Rønning: admired your profound knowledge

Dear Dick,

Lisa and Frode would like to send you greetings from Norway. We are thinking of you, and then many fond memories of time we have spent with you come to our minds. In particular, we remember with pleasure when we came to visit in Madison many years ago, and you showed us around. We went on a tour to look at houses designed by Frank Lloyd-Wright, and we went to second-hand book shops where we got the opportunity to buy some pieces from the Schoenberg collection. And—you showed us the coffee shops where we got take-away coffee.

We have also met in various other exiting places, like Sicily. We think it was then that you and your wife had been to Egypt, and you told us about the horrible traffic in Cairo. Now, we have experienced this by ourselves, and indeed, Egypt is a fantastic country.

We have always admired your profound knowledge about so many different areas of mathematics. You always had insightful comments when someone gave a talk at a conference. And, in addition, your strong interest and engagement for educational matters in mathematics! It is not obvious, but certainly of tremendous importance that mathematicians with your standing, engage in educational matters, from school to university.

We will always remember you, with great respect and pleasure.

Kind regards,
Lisa and Frode

Lisa Lorentzen speaking (with Christian Krattenthaler moderating) at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013.

From: Daniel W. Lozier (d.w.lozier@gmail.com)
Contributor: Daniel W. Lozier: elicited a spirited and useful interaction with the audience

I was fortunate to have the opportunity to be involved in the project to develop a worthy successor to the old Abramowitz and Stegun handbook [2]. It was a mammoth project with many very distinguished participants and taking more than a decade to produce the first public release of the DLMF [59]. Chief among them was Dick Askey. I know I am speaking for all of the editors, authors, validators, and subject area experts who contributed to the success of the project when I say "*Thank You Dick*" for your early enthusiastic support when the project was just a gleam in some of our eyes at NIST, for your insightful advice during the active period of development, for your service as an DLMF Editor, and for your co-authorship of Chapter 1 on Algebraic and Analytical Methods, Chapter 5 on the Gamma Function, and Chapter 16 on Generalized Hypergeometric Functions and Meijer G-Function.

Let me say a few words about Dick's role in the history of the project. In the 90's there began to be increasing recognition of the potential of the web as a potent new way for organizing and disseminating scientific information. Abramowitz and Stegun, and also the Bateman volumes [67, 68], which were the principal handbooks on special functions, were badly in need of serious updating. Their content stretched back nearly a half century. NIST had already begun considering the possibility of a new project, and efforts were underway elsewhere to deal with Bateman. Dick took the important step of co-organizing a two-hour minisymposium (with Willard Miller) at the SIAM annual meeting in 1997. It was titled "*Handbooks for Special Functions and the World Wide Web*". I spoke on the NIST effort[1], and Mourad Ismail spoke on the Bateman effort. Then Dick reflected on the long history of handbooks of special functions. The final 30 minutes were devoted to a general discussion concerning the question of how handbook projects might be pursued and funded. According to my recollection, Dick elicited a spirited and useful interaction with the members of the audience.

Active development began in earnest in 1999 with receipt of NSF funding for the project, and in early 2000, the first meeting of the DLMF Editorial Board took place[2]. Of course, Dick was present and participated actively. Under Frank Olver's direction, who served as Editor-in-Chief and Mathematics Editor, work proceeded steadily. However, anyone who has worked with Frank will know of his meticulous attention to detail, both mathematically and grammatically. He communicated with authors and editors, and later the validators, in voluminous written correspondences by parcel post. This led to an excellent product, as could be expected, but also caused various target dates to be overrun. Fortunately, NSF and NIST were lenient in allowing missed deadlines. The initial release didn't occur until May 2010! During this time Frank often called upon Dick for advice on issues of mathematical, organizational, and presentational nature, and Dick was always generous in his responses.

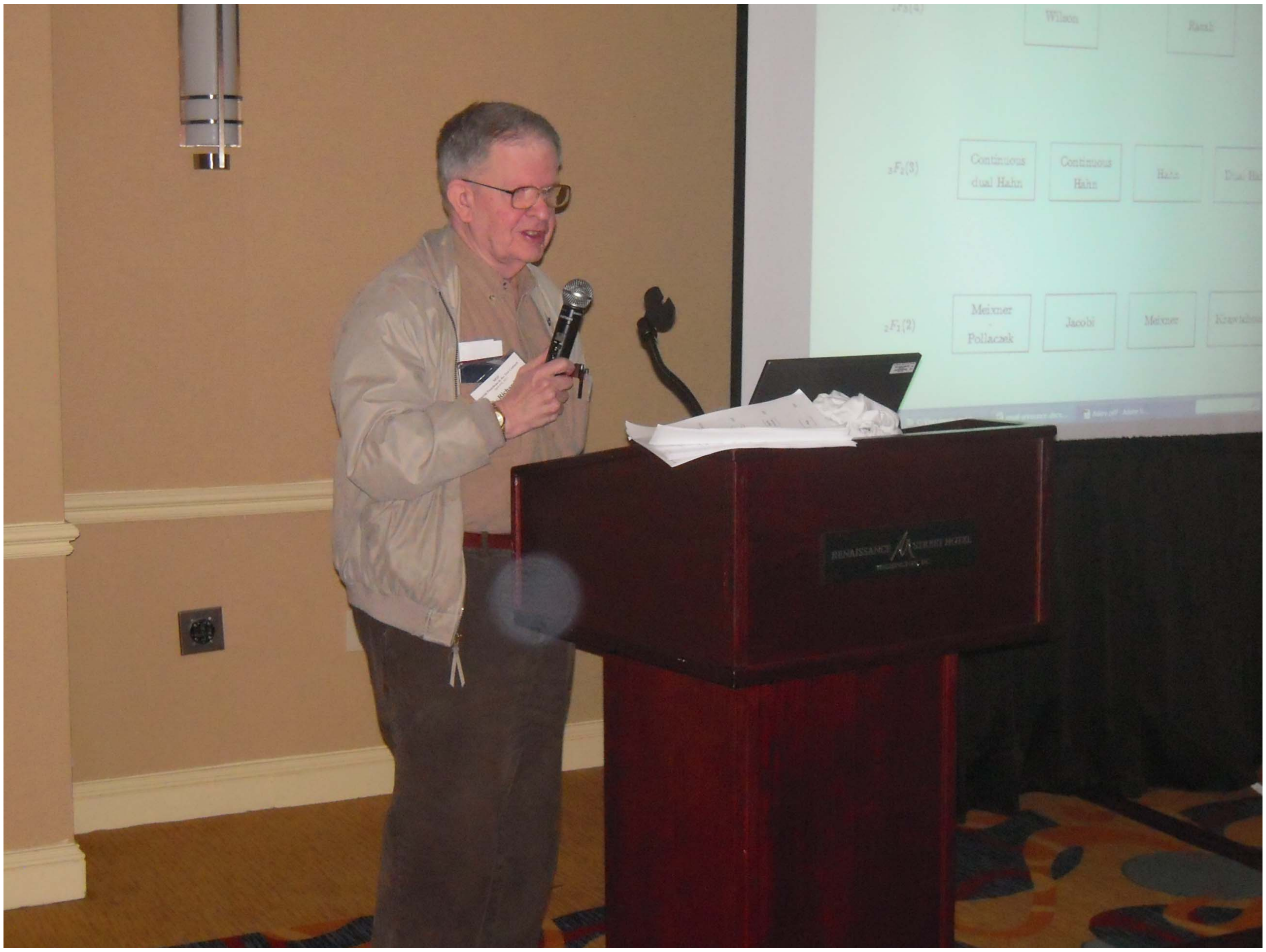
About a year later, in April 2011, a conference, *Special Functions in the 21st Century: Theory and Application* was held in Washington to celebrate the appearance of the DLMF. This conference was dedicated to Frank Olver for his service as Mathematics Editor for the

[1] My presentation provided an account of NIST plans for a possible project. For the written version, see my report [92].

[2] See the contribution by Ron Boisvert (Contribution 9) in this Liber Amicorum for group photos of editors at two meetings in 2000 and 2002.

DLMF. Dick was again front and center as a Plenary Speaker, cementing his key role in the project from inception to first release and beyond.

In conclusion, it is gratifying that the DLMF is having worldwide impact among mathematicians, scientists, engineers, and others who use special functions in their daily work. For this success, we can all be grateful to Dick for his valuable support in so many ways.



This photo was taken at Dick Askey's plenary lecture, in the *Special Functions in the 21st Century: Theory and Application* conference in recognition of Frank Olver and the DLMF, Washington, DC, April 2011. The photographer was Francesco Mainardi.

Contribution #49 ——— Liber Amicorum ——— Dick Askey

From: Doron Lubinsky (lubinsky@math.gatech.edu)
Contributor: Doron Lubinsky: laid the ground for so many subsequent developments

Like so many people in orthogonal polynomials, I benefited greatly from the works and insights of Dick Askey. In particular, his work on mean convergence of orthogonal expansions, and on Marcinkiewicz–Zygmund inequalities for quadrature sums was fundamental and laid the ground for so many subsequent developments [18, 26, 38].

I too remember the lectures where Dick Askey seemed asleep, and yet in question time pointed out errors in the middle of the lecture, or asked penetrating questions. The aphorism "*ten minutes of sleep in a lecture is worth an hour of sleep in your bed.*" is often attributed to Dick, and indeed I always mention his name when telling my classes.

The last lecture of his that I attended was at Georgia Tech. It involved both research and K12 maths education—and reflected the breadth of his interests. Thank you Dick for all you did. You are a model for all of us.

Contribution #50 ——— Liber Amicorum ——— Dick Askey

From: Robert S. Maier (rsm@math.arizona.edu)
Contributor: Robert Maier: otherwise have remained unchanged, if not embedded in amber

I can attribute to Dick Askey, with gratitude, several of the mathematical directions which I am now exploring. I should say first that his service over the past several decades to the fields of special functions and orthogonal polynomials has been immense. As someone who has moved gradually from theoretical physics to pure mathematics, I can testify that his influence has stimulated in several communities the understanding and development of mathematical structures that would otherwise have remained unchanged, if not embedded in amber.

It was by reading his "A look at the Bateman project" [22], a contribution of his to the 1994 Festschrift for Wilhelm Magnus [1] that I first came to understand the quadratic and cubic transformations of higher hypergeometric functions. This has led me to invent new ones, and also to investigate quartic transformations, including quartic transformations that are not products of quadratic ones. And by pondering transformation formulas involving hypergeometric functions that are not only well poised but "*very well poised*" (a most useful, long-needed phrase of his invention!), I have been led to develop additional formulas of the same type [95]. On this topic, his kind comments to me at professional conferences have been greatly appreciated.

Dick Askey's contributions to the analytic side of special function theory have been especially substantial. The usefulness to approximation theory of the polynomials on the upper levels of the Askey and q-Askey schemes is very clear, and the construction of extended and parallel classification schemes is now a topic in its own right. I should add that some of us, having backgrounds in classical analysis but having been intrigued by the Askey-Wilson polynomials and the q-Askey scheme, are now beginning to catch what is cheerfully called the q-disease.

Robert S. Maier, Professor of Mathematics & Physics, University of Arizona, and Adjunct Professor of Mathematics, University of Colorado

Contribution #51 ——— Liber Amicorum ——— Dick Askey

From: Leonard Maximon (max@gwu.edu)
Contributor: Leonard Maximon: happy to share information and knowledge

As a fellow contributor to the DLMF [59] I came to appreciate that Dick Askey was an encyclopedia of information on every aspect of the subject of special functions of mathematical physics. But more than that, whenever I needed help, for a reference, for an explanation of a fine point in the theory, he was open to all questions, gracious in giving assistance, happy to share information and knowledge. It was a privilege to have known him as a colleague.

Contribution #52 ——— Liber Amicorum ——— Dick Askey

From: Willard Miller, Jr. (mille003@umn.edu)
Contributor: Willard Miller, Jr.: Dick Askey helped me in many ways

Dick Askey has helped me in many ways, including baby-sitting for my children at a workshop in the early 1970s, putting in a good word for me with NSF long ago, and writing an introduction for one of my books. He alerted me that his student Dennis Stanton was on the market and hiring him in the 1980s was one of the best decisions of my life. His ground-breaking work on the Wilson and Askey-Wilson polynomials has inspired much of my recent work on superintegrability. I certainly wish him well.

Willard Miller, Professor Emeritus, CSE Distinguished Professor
School of Mathematics, University of Minnesota
127 Vincent Hall, 206 Church St. SE
Minneapolis, MN 55455
612-624-7379, FAX 612-626-2017,
http://www-users.math.umn.edu/~mille003/

Willard Miller, Jr. and Dick Askey

Contribution #53 —— Liber Amicorum —— Dick Askey

From: Stephen Milne (milne@math.ohio-state.edu)
Contributor: Stephen Milne: a true inspiration to me my entire career

My wife Jane and I greatly appreciate Dick's friendship, concern, and support all these years. Both his great character as a person, and brilliant mathematics has been a true inspiration to me my entire career. Dick's mathematics, books, and key support, including job recommendation and related letters on my behalf during my whole career, made a substantial, real positive difference to me and Jane. Moreover, Dick has profoundly advanced and communicated the Mathematical area of Special Functions and it's applications to the broader global mathematical and scientific communities. His contributions here are truly timeless and will endure.

Stephen Milne

Contribution #54 —— Liber Amicorum —— Dick Askey

From: Victor H. Moll (vhm@tulane.edu)
Contributor: Victor H. Moll: first of all, the name is Dick

I came late to the area of Special Functions. Having found a formula for a definite integral I volunteered To give a talk and I called it "How to prove a formula for the Jacobi polynomials when you never heard of it". Dick was in the audience. After the talk he came to me, told me that my talk was nice and then proceeded to tell me what I should learn.

When I came back to New Orleans, I realized that I had forgotten to take notes of our conversation. I wrote him a letter (yes a letter) that started "Dear Prof. Askey". He replied "*First of all, the name is Dick.*" Then he proceeded to write to me about hypergeometric functions. It was great help at a turning point in my career.

Victor H. Moll

From: Paul Nevai (paul@nevai.us)
Contributor: Paul Nevai: Dick and I have a deep secret

Dick and I were quite close to each other. There are just way too many stories both of professional and personal nature that need to be mentioned, or else will be lost forever. Because of this, I plan to write about Dick in the Journal of Approximation Theory and I am sure this project will take a long time to complete. I did a similar project with Carl de Boor about George Lorentz and that took a couple of years to accomplish (but it was well worth the time spent on it).

Dick Askey with George G. Lorentz at USC in 1988.

Let me just mention that Dick was instrumental in my immigrating to the US. I came as a legal immigrant and had my green card even before I crossed the border. It was quite a complicated and long process 45 years ago (legal immigration is not for the faint-hearted). Dick organized almost the entire Wisconsin political apparatus to support my immigration application. Even Senator Proxmire wrote on my behalf (although he mistakenly referred to me as a physicist).

I am not sure when I met Dick first in person because it was before the era of electronic record keeping, but I know that it was when Dick gave a series of lectures on special functions at AMI (today it's called Renyi Institute). It was in 1972 or 1973 or something like that. What I definitely remember is that I was thinking that this Yankee speaks so fast that I understand not a word. Much later I realized that Dick actually speaks at a normal speed although he retained his Missouri accent ("w" is "doubleya").

I want to mention a couple of things before they are all forgotten and lost.

First, Dick and I have a deep secret that for some reason (I don't see why) we decided to keep such. Namely, we had a good reason to believe that one of our distinguished OPs guys was a serial philanderer. No, my reader, don't panic, we won't out you; this particular person has been long dead.

Dick Askey sleeping in 1935.

Second, my first mathematical encounter with Dick, was that I discovered a serious error in one of his papers that put me on the road to OPs. I think I never mentioned this to Dick. The paper was about mean convergence of Lagrange interpolation [17] and the error was the primary reason I became involved in it (because I wanted to find a different approach). However, later I found out that what I saw was just a preprint (an MRC Technical Summary Report) and the final version [18] no longer included the erroneous conclusion. Someone had good eyes before me.

Third, it was a tremendous honor for me when I was asked to write a letter of recommendation for Dick, about 30 years ago when he was considered for membership in the National Academy of Sciences (NAS). However, and this is something I never told Dick before, I lobbied heavily in the 1990s so Dick would be elected to foreign membership of the Hungarian Academy of Sciences but I miserably failed in my mission. The only outcome was that Dick was invited to give the Turán lectures.

Another interesting story is that once Dick and I conspired in something that probably never happened in math before and will never happen again. Namely, once I wrote a paper I didn't really like and submitted it to a prestigious journal where Dick was the editor. Then I asked Dick to pick me as referee and he did and I rejected the paper. Now can anyone beat a story like this? Yes, there was a reason for this seemingly meaningless exercise, but my lips are sealed until 2068.

I love you Dick!
Paul Nevai

Dick Askey getting a shoe shine in front of the Catedral Metropolitana de Santiago while posing as a “rich American capitalist” during a private tour of Santiago, Chile—while attending the *Computational Methods and Function Theory* conference in Valparaíso, Chile, March, 1989. The photo was taken by Paul Nevai.

Dick Askey sleeps during a lecture by Mourad Ismail in 1998.

Mourad Ismail, Dick Askey, Ted Chihara and Paul Nevai in 1998.

From: Peter J. Olver (olver@umn.edu)
Contributor: Peter Olver: brilliance, insight, enthusiasm for mathematics, and strong opinions

Given their mathematical proximity, I wish my father were still around to write this encomium to Dick. They were the best of friends—fellow toilers in the special function vineyards—despite their distinct styles and mathematical tastes, and despite not always seeing eye to eye. Dick is more the wild American, a devotee of the luxuriant jungle of the Bateman manuscript (although of course Bateman himself was originally British), while my father, being the true Englishman that he remained despite moving to the US in 1961, preferred the meticulously cultivated garden of Abramowitz and Stegun and, of course, the subsequent DLMF [59]. Nevertheless, Dick contributed to three of the chapters of the DLMF one of which he even coauthored with my father, Ranjan Roy and Roderick Wong—which was, in fact, their only collaborative effort. Also, I would not be surprised if Dick were the one who convinced my father to include the chapter on Meijer G-functions, which Dick co-wrote with Adri Olde Daalhuis, since my father was of the opinion that they were much too general to be of any use. (However, right after he died, I happened to ask Mathematica to evaluate a certain integral involving the Airy function, and was surprised when an answer came back in terms of a hypergeometric function. I subsequently discovered that the package that accomplished this was in fact based on Meijer G-functions. Sadly I was no longer able to get my father's opinion on this development.) Further, while certainly acknowledging their importance, my father was far less into orthogonal polynomials than Dick, preferring to concentrate his efforts on Airy, Bessel, (confluent) hypergeometric, Whittaker, and the like.

As a result of their connection, I knew Dick well before starting my own mathematical career, although my memory of when and how I first met him is cloudy. I do vividly remember the famous conference in Wisconsin in early spring, 1975, when I was still in graduate school, and where they were both in attendance and in top form, with many disagreements, and finally ending in an epic snowstorm that closed off the airport when we were due to depart. (Somehow this did not dissuade me from eventually going to Minnesota!) While I learned enough about special functions to use them on those rare occasions they showed up in my own work, I did not follow their lead (or that of my colleagues Willard Miller and Dennis Stanton, for that matter), preferring to toil in my own (somewhat unkempt, but more "symmetrical") mathematical garden. I, of course, ran into Dick on many occasions, particularly when returning to Wisconsin, and was always in awe of his brilliance, insight, enthusiasm for mathematics, and strong opinions on many subjects, that he did not hesitate to state. And, while I am not close mathematically, I still regard him as one of my early inspirations as to what it would be like to be a true mathematician.

So both my wife Cheri (another mathematician, as is our son Sheehan) and I wish Dick the very best on this occasion. I wish I could be there to celebrate in person, but my continuing administrative duties and other travels are tying my hands at this time.

All the best, Dick!
Peter

From: Ken Ono (ken.ono.691@gmail.com)
Contributor: Ken Ono: the significant roles you played in my life

For Dick,

It was a pleasure to speak at the lovely retirement dinner in your honor in Madison many years ago. Although I no longer have my notes, I remember some of the thoughts I shared with our colleagues at the time. You have been a gift to mathematics and mathematicians (of all ages). Let me remind you of the significant roles that you have played in my life.

I first learned of Ramanujan thanks to the worldwide effort that you spearheaded in the early 1980s. My father was one of the many mathematicians who made a contribution for the Granlund bust, and he treasured the thank-you note that he received from Janaki Ammal. This note has served as a constant reminder of beauty in mathematics, and is now one of my most cherished possessions. In fact, the note is prominently displayed on the wall in my office, and it welcomes me each work day. Thank you.

I have many fond memories of your lectures on special functions, orthogonal polynomials, and their role in number theory. The flair with which you introduced Ramanujan's identities with your t-shirts was breathtaking. I wish we had captured one of these lectures on video so that young students entering the field today can share the wonder of these beautiful identities. Thank you.

Your encyclopedic knowledge of mathematical history was mind-blowing. The commentary you added at the end of lectures made us recognize the roles we played in our own research. Instead of concentrating on grants and numbers of papers, I learned from you that important research are like bricks. An important theorem has to fit well with the edifice of theorems laid before us by great mathematicians. This realization was important to me during my formative years. Thank you.

In addition to your role as a world class mathematician, I have to thank you for your investment in education. I remember the days you would read Dr. Seuss to your UW math education students. I thank you for taking a stand on critical issues which plague K-12 education. Although the challenges remain, I am happy to report that your voice persists (certainly among the AMS leadership and the US National Committee for Mathematics). I am doing my best, as are many others. Thank you.

For so many reasons you have been a gift to mathematics and mathematicians.
Thank you.

With the deepest admiration.
Ken Ono

From: Peter Paule (ppaule@risc.uni-linz.ac.at)
Contributor: Peter Paule: the writer must be a leader in the field!

My personal "discovery" of Dick Askey went via George Andrews. Around 1980, inspired by Johann Cigler's seminars on q–series, I was led to Andrews' thirty pages article "Problems and Prospects for Basic Hypergeometric Functions" [9]. It was part of a proceedings volume and, before producing a xerox copy of Andrews' article, I glanced at some other entries there. In particular, the preface written by the editor caught my attention. It started like this:

> *Twenty years ago it was widely believed that the existence of large, fast computing machines spelled the end of the study and use of special functions. Differential equations would be solved numerically, integrals would be evaluated numerically, and special functions would become a fossil. Many mathematicians acted on this belief, and special functions disappeared from their traditional place in the curriculum as part of a course in complex variables. They were often replaced in the mathematical physics course by a section on Hilbert space and functional analysis.*

It is not too difficult to guess that the writer of these lines was Dick Askey who was editing this volume containing the proceedings of the Advanced Seminar on Special Functions held in Madison, Wisconsin, March 31 to April 2, 1975. Askey continued his preface as follows.

> *To paraphrase Mark Twain, reports of the death of special functions were premature. A sociologist of science would accept the following two facts as proof. Over 150,000 copies of the Handbook of Mathematical Functions have been sold. Two of the five most widely cited mathematics books (as measured by the citations in Science Citation Index) are this book and Higher Transcendental Functions by A. Erdély at al. Most of us are more concerned with quality than quantity, and so would not be convinced by these two facts. A more compelling proof is given in this book. Some interesting results, conjectures, and problems are given. In addition there are surprising connections with other fields.*

This description of the state of the art was followed by a short but brilliant summary of the contributions to the book. To make it short: Askey's preface impressed me quite a bit. It made clear to me that the writer must be a leader in the field!

Now, more than 40 years after this Madison meeting, also Askey's quality as a prophet is proven by the impressive evolution of the field of special function. To add some data in the spirit of a sociologist of science: the *15th International Symposium on Orthogonal Polynomials, Special Functions and Applications (OPSFA)*, organized by RISC (Kepler University Linz, Austria) and the Radon Institute (Austrian Academy of Sciences) in Hagenberg (July 22 to 26, 2019, Austria) saw more than 200 participants. At this conference, in recognition and appreciation for his outstanding work and leadership in the field of special functions, a Lifetime Achievement Award has been presented to Dick Askey; see the picture attached.

The first time I met Dick in person was the *NSF-CBMS Regional Research Conference on Special Functions, Physics, and Computer Science* at the Arizona State University in May 1985. In addition to the organizers, Mourad Ismail and Ed Ihrig, Askey played a visibly important role at this meeting—which fully confirmed my first impression when reading his preface as a student. Later I had the pleasure to enjoy scientific exchanges with Dick

Dick Askey's Lifetime Achievement Award.

at numerous conferences; in addition, I have fond memories of meeting him at various editorial meetings in connection with the Digital Library of Mathematical Functions (DLMF) [59], the NIST successor project to the above mentioned Handbook of Mathematical Functions. At these occasions I always felt that Dick Askey is following a strong commitment to unearth the essence of mathematical concepts and thoughts.

Last but not least, I want to thank him cordially for his strong support at various occasions. For example, to arrange with many other obligations, he took the pain of going from

Madison to Austria for two days only: he wanted to be personally present at an important decision meeting organized by the Austrian Science Foundation! A more relaxed trip to Austria was his visit of RISC in August, 2011; see the picture.

Dick Askey with staff at RISC in 2011.

Dear Dick,
Thank you very much for inspiration and support, and all best wishes to you and your family.
Peter Paule (September 10, 2019)

From: Veronika Pillwein (veronika.pillwein@risc.jku.at)
Contributor: Veronika Pillwein: I had the feeling, he will see right through the result

The theme for my PhD thesis was designed to be at the interface of Symbolic Computation and Numerical Analysis by my supervisors Peter Paule and Joachim Schöberl. Naturally, the book that was useful the most—and visibly suffered the most—was the red book "Special Functions" by George Andrews, Dick Askey, and Ranjan Roy [14]. In particular it provided the keys for proving an inequality over Gegenbauer polynomials conjectured by Joachim for the Legendre case.

When I met Dick Askey during his visit at RISC in 2011, I had the feeling, he will see right through this result—and so he did. He gave a simple and elegant proof of Joachim's original conjecture. Beyond that, he has a point of view on the ultraspherical case that certainly shows the right way to extend the positivity result to general Jacobi polynomials. He was sharing this insight in his gentle way, telling anecdotes and giving advice on Mathematics and Education at the side.

Dear Dick, I am very grateful for your support and your generous sharing of time and curiosity.
Thank you!

From: Mizan (Mizanur) Rahman
Contributor: Mizan Rahman: I doubt I would ever have any reason to consider the thought

Apart from Dick's encyclopedic knowledge in the field of special functions, which he was always happy to share with anyone interested, his contributions to this much-loved area of his can be broadly divided into two parts, in my opinion.

One: he brought new life into an otherwise stagnating field that mainstream mathematicians were beginning to look with a bit of condescension, perhaps even with a touch of scorn. Dick made it look modern and almost fashionable, because of his gift of communicating the message that special functions is one of the few classical areas where almost all great mathematicians of the past had found much to their liking, and that it is not just an aide to physicists' problems but also connected with many diverse areas like combinatorics, approximation theory, coding theory, probability, group representation theory, and scores of others. In fact, he convinced us, it is a great field on its own.

Two: Dick was able to attract young, talented and ambitious mathematicians to this field, which did not seem to have as much glamor as, say, algebraic geometry but nonetheless became a very attractive and exciting field to spend a whole life-time working in. Suddenly there were special sessions on Special Functions everywhere in the world.

Personally, I owe my career in Special Functions to Dick Askey. Without his encouragement I might still be laboring in the wasteland of statistical mechanics trying to find yet another useless special solution of the Boltzman equation. He rescued me from that state of futility and introduced me to a bright new world of beautiful ideas and exciting problems where brilliant young mathematicians like Tom Koornwinder and George Gasper were discovering great new results. I was completely sold. Without Dick, I doubt if either George or I would ever have any reason to consider the thought of writing a book on q-series (see [77]).

Thanks for everything, Dick.
Mizan

Collected from an email to Diego Dominici

l to r: Jasper Stokman, Mizan Rahman and Dick Askey at the *International Workshop on Special Functions*, Hong Kong in June 1999

From: Donald Richards (dsr11@psu.edu)
Contributor: Donald Richards: a tall smiling man approached me and introduced himself

In October, 1982 I gave a talk at the AMS regional meeting at College Park, MD. My talk was on the Bessel functions of matrix argument, and I was nervous, for I was sure that the audience (which included Biedenharn, Dunkl, Faraut, Gross, Holman, Koornwinder, and Terras) knew more about Bessel functions than I did.

After my talk ended, a tall, smiling man approached me and introduced himself as Dick Askey. By the end of the meeting, Dick had invited me to visit him at Madison, which I did in February, 1983. I always recall that Dick was the first ever to invite me to lecture at another school, that he eagerly introduced me to I. J. Schoenberg, and that he arranged for me to visit Dennis Stanton at Minnesota.

Dick introduced me to Selberg's integral, a topic on which I now have several papers, and he suggested that I study the area of q–series, which I declined to do because "q–series are too difficult for me." Dick then noted that "some q–hypergeometric series are special cases of the hypergeometric functions of Hermitian matrix argument, so the problems that you and Gross are studying are more difficult than some q–series problems." I realized later that he saw more promise in me than I saw in myself.

On the other hand, Dick did get me to look at Ramanujan's Notebooks [107]. That later motivated Ding, Gross, and me to generalize Ramanujan's Master Theorem to the setting of symmetric cones, a result which has spawned numerous papers by others and which has also led to generalizations of the Cauchy–Frullani integral.

Dick told me some remarkable stories about his experiences outside of research. My favorite is the story of a trip home from a conference in Moscow, Russia, during the era of the Soviet Union. Dick explained that, due to a modified flight schedule, he had to fly from Moscow to Edmonton, Alberta with the intention of taking a connecting flight to the U.S. When Dick arrived in Edmonton, dressed in a conspicuous Russian–style winter hat with prominent ear flaps, he found himself being viewed suspiciously by Canadian security officials. The RCMP thought that Dick, dressed in his usual rumpled style, might be a Soviet agent. After phone calls were made to someone (Al-Salam?) at the University of Edmonton, Dick was allowed on his way.

In October, 2009 Dick attended the AMS meeting at Penn State University, and my wife and I invited him to dinner at our home. During dinner, I mentioned to Dick that the Institute of Jamaica, which is based in Kingston, Jamaica, had awarded my wife in 2008 the Institute's Musgrave Gold Medal for her work in astronomy, and that I was impressed by her award. At that point, Dick chided me gently that I ought to be far, far more impressed than I appeared to be. Dick explained that there had been a linguist at Wisconsin who was awarded the Institute's Musgrave Gold Medal, but that recipient had been in his mid–70's when he received his award, whereas my wife was then in her early 50's. My wife and I were stunned: This guy, Askey, knew more about the Institute of Jamaica than us Jamaican–born!

Looking back on the years that I have known Dick, it is impossible for me to thank him enough. He has been a mentor, adviser, and friend. His kind words of condolence three years ago were heartfelt by my daughters and me. His comment in 1982 that my talk at College Park was well–received by the audience constituted enormous encouragement to someone then trying to find their niche in research; and even today, 37 years later, I continue to publish on the Bessel functions of matrix argument.

Dick, I send my deepest gratitude.

From: Hjalmar Rosengren (hjalmar@chalmers.se)
Contributor: Hjalmar Rosengren: you should look at the ${}_{10}W_9$!

Let me just share a story which is in line with many others.

The first time I met Dick Askey was at the Mt. Holyoke meeting in 1998 on the occasion of his 65th birthday. At that time I was a young PhD student who had just fallen in love with q-series, a topic that nobody else at my place had any interest in. This was actually the first time I met many people in the field, and it was wonderful to discover how friendly and welcoming they were to a newcomer.

One slight disappointment was that the day I gave my talk, Dick had to attend some important hearing on mathematics education. The day after that, he came up to me and said "*We need to talk.*" We had a chat and he gave me a strict order: "*You should look at the 10-W-9!*". These words really had a huge impact on my career. Properly understanding the 10-W-9-function was a key for much of my later work; in particular, it led me into the exciting field of elliptic hypergeometric functions. Thinking back of it, it amazes me that Dick had taken the time to question people about the talks he had missed and based on the answer somehow found exactly the right words to guide a completely unknown student. I have heard very similar stories about Dick from many friends. His special ability must come from combining a profound understanding of special functions with an equally deep understanding and care for other people. I am very grateful for the interactions I had with Dick during my career.

Best wishes, Hjalmar Rosengren

Dick Askey's 80th Birthday Conference in Madison, Wisconsin, USA in 2013.
The picture was taken by Patsy Wang-Iverson.

From: Ranjan Roy (royr@beloit.edu)
Contributor: Ranjan Roy: Euler's evaluation of a certain hypergeometric series

Euler's Evaluation of a Certain Hypergeometric Series
Ranjan Roy
September, 2019.

Dick Askey's main focus is and has always been the advancement of mathematics, and thus of mathematicians. He has helped us in many ways: for example, by bringing together mathematicians with complementary interests; by conducting seminars and workshops on new developments and results; or by using his unique insight to guide us in profitable mathematical directions.

Dick has also been able to teach us that a deep understanding of received mathematics can be a key step toward new discoveries. His mathematical sagacity and his fresh insight into well-known results are illustrated in his evaluation of Euler's beta integral, given on pages 5-6 of his *Special Functions* [14]. In his new and revealing evaluation, only slightly different from Euler's, Dick obtained an interesting and novel approach to a well-established topic.

There are numerous examples of Dick's mathematical perceptiveness, especially in connection with hypergeometric series; many of these are contained in the treasure trove of his unpublished lecture notes.

Last year, I saw a paper of Euler in which he had converted a certain hypergeometric series into an infinite product. I immediately wondered what Dick might have done with this example.

Euler's example occupies a paragraph in his 40-page paper, "De mirabilibus proprietatibus unciarum, quae in evolutione binomii ad potestatem quamcunqua evecti occurrunt" [71, Eu. I-15, 528-568], presented to the Petersburg Academy in 1776, when Euler was 69 years old. Euler stated the formula

$$1+\binom{n}{1}\binom{n'}{1}+\binom{n}{2}\binom{n'}{2}+\binom{n}{3}\binom{n'}{3}+\cdots=\frac{1}{n\int_0^1 x^{n'}(1-x)^{n-1}\,dx}. \tag{9}$$

Euler was unable to prove this formula, writing that he found it remarkable that there was no direct general proof. He verified the formula for a number of particular cases. In the case for which $n'=-n$, note that the right-hand side equals

$$\frac{1}{\Gamma(1+n)\,\Gamma(1-n)}=\frac{\sin n\pi}{n\pi}.$$

But Euler took a different approach to (9) for the case $n'=-n$; here note that Euler implicitly assumed that $0<n'<1$. He transformed the integral to

$$\int_0^\infty \frac{y^{n'-1}\,dy}{1+y},$$

the value of which he knew from his 1742 work on the integration of rational functions: $\frac{\pi}{\sin n'\pi}$. Thus, in Section 44 of his paper, he found that he needed to verify the formula

$$1-\frac{n^2}{1^2}+\frac{n^2(n^2-1)}{1^2\cdot 2^2}-\frac{n^2(n^2-1)(n^2-2^2)}{1^2\cdot 2^2\cdot 3^2}+\cdots=\frac{\sin n\pi}{n\pi}, \tag{10}$$

where the series on the left-hand side was obtained by taking $n'=-n$ in the series in (9). To verify (10), Euler denoted the left-hand side by S and observed that $1-n^2$ was a common factor of the sum. Dividing by this common factor, he arrived at

$$\frac{S}{1-n^2}=1-\frac{n^2}{2^2}+\frac{n^2(n^2-2^2)}{1^2\cdot 2^2\cdot 3^2}-\frac{n^2(n^2-2^2)(n^2-3^2)}{1^2\cdot 2^2\cdot 3^2\cdot 4^2}+\cdots. \tag{11}$$

Askey and Roy at Beloit in 1997 $-+-$ Roy and Askey at Madison in 2019.

One can now see that $1-\frac{n^2}{2^2}$ is a common factor of the sum in (11). So, rewrite the sum in (11) as

$$\frac{S}{(1-n^2)(1-\frac{n^2}{2^2})}=1-\frac{n^2}{3^2}+\frac{n^2(n^2-3^2)}{1^2\cdot 3^2\cdot 4^2}-\cdots,$$

where $1-\frac{n^2}{3^2}$ is clearly the common factor. Repeating this process infinitely often, Euler found that

$$\frac{S}{(1-n^2)(1-\frac{n^2}{2^2})(1-\frac{n^2}{3^2})\cdots}=1$$

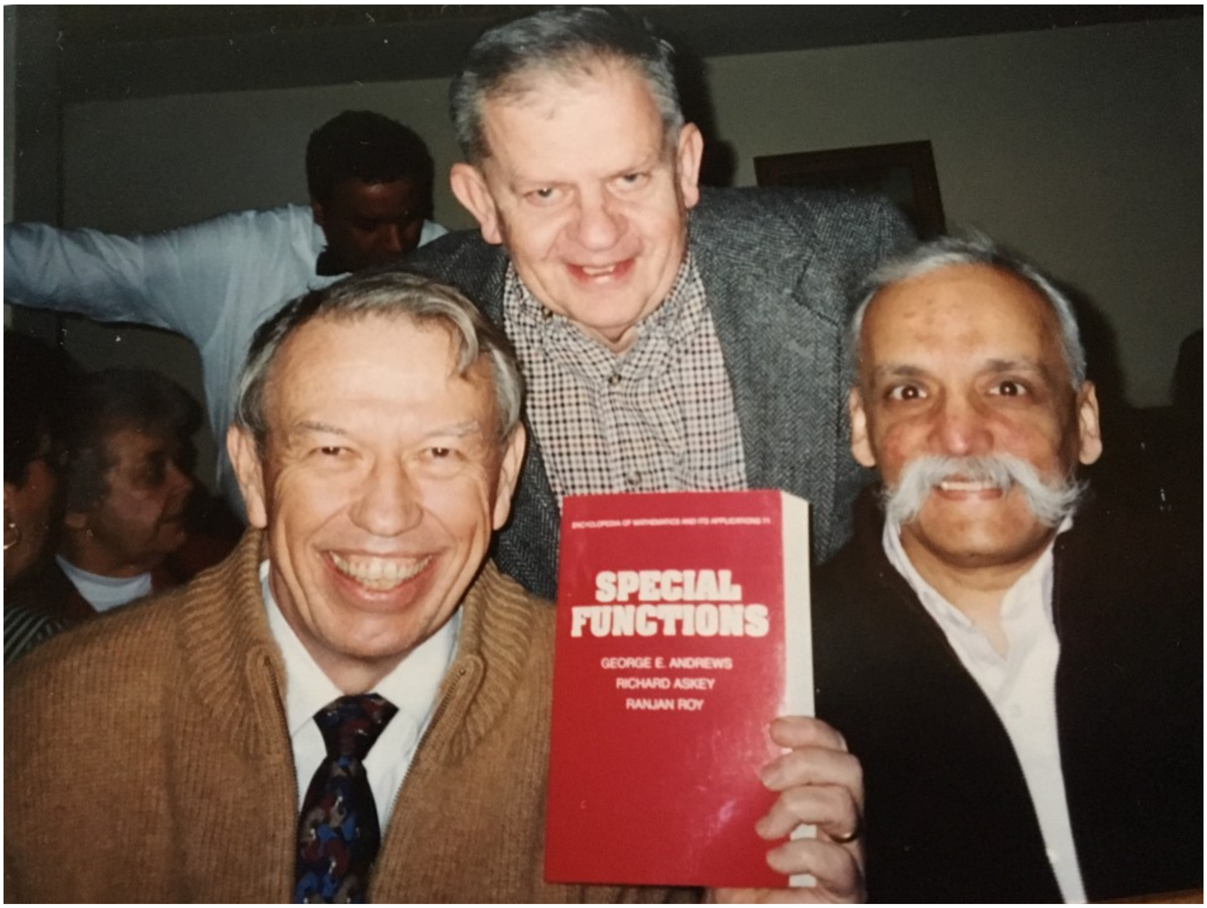
and, since the product was $\frac{\sin n\pi}{n\pi}$, Euler had succeeded in verifying (9) for $n'=-n$. Of course, (9) is a particular case of Gauss's sum for $F(a,b;c;1)$, but Euler's method of verifying (10) by converting a series into a product is of great interest.

In a June 1750 letter to Goldbach [69], Euler deduced the pentagonal number theorem, starting with the observation that

$$\begin{aligned}&(1-\alpha)(1-\beta)(1-\gamma)(1-\delta)\cdots\\ &\qquad=1-\alpha-\beta(1-\alpha)-\delta(1-\alpha)(1-\beta)-\gamma(1-\alpha)(1-\beta)(1-\delta)-\cdots.\end{aligned} \tag{12}$$

Euler's example of the conversion of a hypergeometric series to a product is a particular case of (12), although he started with the right-hand side. Thus, Euler's idea can be applied to series other than the hypergeometric series, in particular q-series. Gauss gave examples of this method in his work on theta functions.

As I considered Euler's examples and (12), I was inspired by Dick Askey's teachings.



Andrews, Askey and Roy at Baltimore in 2003.

From: Michael J. Schlosser (michael.schlosser@univie.ac.at)
Contributor: Michael J. Schlosser: thanks a lot for serving as an idol and inspiration!

A tribute to Dick Askey
A small contribution to the September 2019 Richard Askey Liber Amicorum
Michael J. Schlosser

Dick Askey has always been very supportive to the mathematical community in general, and in particular to his mathematical family: the Special Functions community which he cared about and steadily nurtured (like a Godfather, but completely gently and nonviolently!). I wish to thank him for his support and advice over the years (during which he several times gave me encouraging feedback on my work, was writing recommendation letters when being asked, was giving me specific valuable advice on old literature to read—for instance, by Heinrich August Rothe [113], and by Ferdinand Schweins [115]—, and so on).

It is difficult to estimate the value or the impact of a single person in one's life. Dick Askey has had a steady light impact on me (just like water which is inconspicuously dripping but is ultimately creating a canyon by erosion). He was already one of the great shots in Special Functions when I started to study Mathematics at the University of Vienna, so for me he has always been around (and I took that for granted, I never knew anything else). It was in June 1996 (I just had completed my PhD thesis, working under the direction of Christian Krattenthaler on multivariate basic hypergeometric series) when I first met Dick, namely at the *Miniconference on q-Series* which Gaurav Bhatnagar and Stephen Milne organized at the Ohio State University. It was at this meeting where I first personally experienced Dick Askey in action promoting special functions and supporting young people. There were several other occasions where I was lucky to attend the same conference where Dick was, including a conference on Ramanujan in Mysore, India, in December 2012, to single out a somewhat (for me) exotic place where we met as well.

I now want to turn to mathematics, first abstractly, then concrete. On this occasion of compiling a contribution to the Richard Askey Liber Amicorum, I would like to offer an (at least small) piece of mathematics to Dick as a gift, as a (small) token of appreciation. Various keywords come to (my) mind while contemplating about Dick Askey and his work. *Special functions* and *q-series* definitely belong to the main keywords. The imperative "*Read the masters!*" certainly comes to one's mind as well. Dick has always stressed the importance of reading the work of the old masters (Euler, etc.) for a better understanding of mathematics (or in the creative art of actually doing mathematics), and argued that this would also have a strong impact on one's research ability, either by discovering what the old masters already knew or just by learning from their thought process. Last but not least, *positivity problems* and *problems of analytic flavor* are intimately connected to Dick Askey's work too. A mathematical gift to Dick should ideally connect various of the keywords just mentioned. (I admit that I have left out "*orthogonal polynomials*" but allow me a certain degree of artistic freedom to justify my—albeit arbitrary—thought process!)

Consider the well-known binomial theorem,

$$(1+z)^n = \sum_{k\geq 0} \binom{n}{k} z^k, \tag{13}$$

with the binomial coefficient defined by

$$\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{k!}.$$

In (13), the exponent n is a priori a nonnegative integer. Isaac Newton, one of the old masters, experimented with this identity, formally replaced the exponent n by some fraction such as $\frac{1}{2}$, etc., and showed that the identity still holds when both sides make sense. (Today we know that in the nonterminating case, i.e., when n is not a nonnegative integer, z is required to satisfy the condition $|z| < 1$, unless z is considered a formal power series variable.) Newton was thus the first person to consider a "fractional" extension of the binomial theorem. In fact, as we know today, the integer parameter n in (13) can be replaced by any complex number α; the identity then still holds (provided $|z| < 1$, or if z is simply a formal variable).

I would like to dedicate an observation, at this moment actually still a conjecture, to Dick, which can be regarded to be a *fractional* extension of limiting cases of the First and Second Borwein Conjectures (cf. [13]).

Let q be a complex variable with $0 < |q| < 1$. As usual, the q-shifted factorial is defined as $(a;q)_0 = 1$ and

$$(a;q)_n = (1-a)(1-aq)\cdots(1-aq^{n-1}).$$

This also makes sense for $n = \infty$; then the product has infinitely many factors. For convenience, we shall also define

$$(a_1,\dots,a_m;q)_n = (a_1;q)_n\cdots(a_m;q)_n$$

(of which we will only use the $m = 2$ case in this tribute).

The celebrated First Borwein Conjecture (made by Peter Borwein around 1990, see [13]) asserts that for each nonnegative integer n, the polynomials $A_n(q)$, $B_n(q)$ and $C_n(q)$ appearing in the dissection

$$(q,q^2;q^3)_n = A_n(q^3) - qB_n(q^3) - q^2C_n(q^3) \tag{14}$$

are actually polynomials in q with *nonnegative* integer coefficients.

This conjecture was open for a long time and has only very recently been settled by Chen Wang, in his 2019 doctoral thesis at the University of Vienna (supervised by Christian Krattenthaler), see also [130] (which is part of his thesis). Chen Wang's method of proof (which follows a suggestion made by George Andrews in [13]) is analytic in nature and makes careful use of asymptotic estimates to establish bounds on the coefficients.

The First Borwein Conjecture is actually easy to show in the limit case $n = \infty$. In that case one can make use of the famous Jacobi triple product identity to prove the claimed nonnegativity of the series $A_\infty(q)$, $B_\infty(q)$ and $C_\infty(q)$.

The Second Borwein Conjecture (still being open) concerns a similar dissection in terms of powers of q, but with the expression on the left-hand side of (14) being squared.

Gaurav Bhatnagar (whom I know in person as long as Dick) and I have recently formulated partial theta function extensions of the first two Borwein Conjectures, see [52]. There we replaced all the factors in the respective q-shifted factorials by partial theta functions and observed that similar positivity properties appear to hold. My aim here is not to redeliver the results I have obtained with Gaurav but rather to present something entirely new (and unspoiled!):

Conjecture 1 (Dedicated to Dick Askey). *Let q be a complex number with $0 < |q| < 1$ (or view q as a formal power series variable). Further, let d be a real number satisfying*

$$0.22799812734\ldots \approx \frac{9-\sqrt{73}}{2} \leq d \leq 1 \quad \text{or} \quad 2 \leq d \leq 3.$$

Then the series $A^{(d)}(q)$, $B^{(d)}(q)$, $C^{(d)}(q)$ appearing in the dissection

$$(q, q^2; q^3)_\infty^d = A^{(d)}(q^3) - qB^{(d)}(q^3) - q^2C^{(d)}(q^3) \tag{15}$$

are power series in q with nonnegative real coefficients.

(The $d = 3$ case of the above conjecture is the $n = \infty$ case of an observation made by Chen Wang, communicated to the community by Christian Krattenthaler in his recent plenary talk at OPSFA15 in Hagenberg, Austria, in July 2019.)

Dick, I hope you like the above conjecture and can help to settle it!

I wish you many more years of joy and productivity.
Thanks a lot for serving as an idol and inspiration!

Contribution #65 —— Liber Amicorum —— Dick Askey

From: Susan Sclafani (sclafanisusan@yahoo.com)
Contributor: Susan Sclafani: broke down some of the barriers between views

I first met Dick Askey as we began the Mathematics Science Initiative at the U.S. Department of Education in 2001-2002. Patsy Wang-Iverson was an early participant and suggested that we needed to involve Dick as we gathered mathematics professors, researchers, government representatives and practitioners. Dick worked with us for three years as we involved state teams and Title 1 supervisors in planning sessions to improve the quality of mathematics and science education in the country. Because of his involvement, we were able to develop a robust group of professors of mathematics who broke down some of the barriers between views of how mathematics should be taught from Kindergarten through Grade 12. I remain grateful for his participation.

Susan Sclafani
Former Counselor to the Secretary of Education
434 M Street SW, Washington, DC 20024
202 379-6378 Mobile
sclafanisusan@yahoo.com

Contribution #66 —— Liber Amicorum —— Dick Askey

From: James Sellers (jsellers@d.umn.edu)
Contributor: James Sellers: many thanks for your lifelong commitment to mathematics

Dick,

Many thanks for your lifelong commitment to mathematics, especially as it relates to our areas of research, as well as mathematics education. You have been one of the foundational members of our mathematical community for decades, and I am truly thankful for your work which has led the way for so many others to follow. I wish you the very best.

James Sellers

Contribution #67 ——— Liber Amicorum ——— Dick Askey

From: Barry Simon (barry@familysimon.net)
Contributor: Barry Simon: clearly a labor of love for you

Dear Dick,

On behalf of the mathematical community, I should like to thank you for your efforts on putting together Szegő's complete works [122, 123, 124]. Your additions turned what might have been a dry compendium into lively reading and made the document much more valuable. It was clearly a labor of love for you but also immensely valuable for those of us who shared interests with Szegő. So thanks so much.

Barry Simon
Caltech

Contribution #68 ——— Liber Amicorum ——— Dick Askey

From: Alan Sokal (as2a@physics.nyu.edu)
Contributor: Alan Sokal: an inspiring 18^{th}–century mathematician

Dear Dick,

As a total amateur in special functions—but one who has found that they are playing a larger and larger role in my research—I want to thank you for all the inspiration that your work has given to me.

And also to thank you for your patient answers to my e-mail questions, which now go back more than 20 years. One of your responses from 2006 was typical. It began

> *First, address me as Dick, not Professor Askey.*

So here are my thanks to an inspiring "18^{th}–century mathematician" from an aspiring 18^{th}–century colleague [8]

> *Although Dick proudly and jokingly classifies himself as one of the last breed of 18^{th} -century mathematicians ....*

With my best wishes,
Alan

From: Vyacheslav P. Spiridonov (spivp@yahoo.com)
Contributor: Slava Spiridonov: classify the above q–beta integrals

Dear Dick,

I just gave at a conference in Dubna a survey talk on a very fruitful relation between modern quantum field theory and elliptic hypergeometric integrals [121]. Two days before on another occasion, I spoke on how continuous (!) q–deformed Painlevé transcendents and quantum algebras emerge within the standard quantum mechanics through the ideas of self-similarity (Ramanujan integrals show up there through the coherent states). Of course, I had to give a panorama of currently known classical special functions and right in the heart (the center) of that I put the Askey–Wilson polynomials. So, scientifically you are always with me.

In the early days, when I have realized the q–harmonic oscillator algebra on the standard Schrödinger equation you wrote me a letter (probably in 1992 or 1993). Pity that it was lost, but I remember that I said to myself after reading it "*I entered the world of q-special functions which I do not know a lot about.*" Since that time we had more or less regular contact, which faded over the last decade mostly because of my enormous scientific (see above) and personal life business. I would strongly appreciate if you drop me a greetings line, if possible, to spiridon@theor.jinr.ru, similar to what I am doing now.

Here is an entertaining scientific problem that I came to in my recent Adv. in Math. paper [120]. Let us take some positive integer r and the variables $t_1, \ldots, t_5, q \in \mathbb{C}$ are such that $|t_a|, |q| < 1$. Then

$$\frac{(q^r;q^r)_\infty}{2\pi \mathrm{i}} \int_{\mathbb{T}} \frac{(q^{r-1}Az, Az^{-1}, q^{r-1}z^2, qz^{-2}; q^r)_\infty}{\prod_{a=1}^3 (q^{r-1}t_a z, t_a z^{-1}; q^r)_\infty \prod_{a=4}^5 (t_a z, qt_a z^{-1}; q^r)_\infty} \frac{dz}{z}$$

$$= \frac{\prod_{a=1}^3 (At_a^{-1}; q^r)_\infty \prod_{a=4}^5 (q^{r-1}At_a^{-1}; q^r)_\infty}{\prod_{1\le a<b\le 3} (q^{r-1}t_a t_b; q^r)_\infty \prod_{a=1}^3 \prod_{b=4}^5 (t_a t_b; q^r)_\infty (qt_4 t_5; q^r)_\infty}, \quad A = \prod_{a=1}^5 t_a.$$

A similar relation takes place if you flip multipliers q and q^{r-1} in the arguments of q–shifted factorials. I guess these formulae represent a new layer of q–hypergeometric identities which were not considered in the literature before. Clearly they look as scrambled versions of the Rahman integral. I am sure that the full class of such integrals is rather rich and contains more discrete parameters. They originate from $p \to 0$ limit of what I called rarefied elliptic hypergeometric integrals, fully general class of which is not considered yet. However, such q–beta integrals should be classified already at the standard q–level from scratch. And here your method from the paper in "Ramanujan revisited" collection [15] should work. Note that it helped me already to prove the elliptic beta integral (see references in Russ. Math. Surveys 56 (2001) 185 [119]) and it should be modifiable to tackle these cases as well. I hope to spend some time trying to classify the above q–beta integrals in this way, though this is not a favorite task in the list of around 30 problems on my desk.

When I wrote you on 09/11/2001, you replied that you find solace on that day only in supporting letters from friends. So, Dick, I am very happy to send you my greetings with the wishes of health, scientific pride and many friends around for a permanent support.

Slava Spiridonov
Dubna, 12 September 2019

Group photo at *International Workshop on Special Functions, Orthogonal Polynomials, Quantum Groups and Related Topics: dedicated to Dick Askey on his 70th Birthday,*
Hotel Hochwiesmühle, Bexbach, Saarland, Germany, October 18–22, 2003.

Contribution #70 ——— Liber Amicorum ——— Dick Askey

From: Dennis Stanton (stant001@umn.edu)
Contributor: Dennis Stanton: an exciting time to be a student

I was in Dick Askey's Special Functions class, Mathematics 805, in Madison. His first lecture was on August 27, 1974. Also in that class were his students Dan Moak and Jim Wilson, and Mourad Ismail was there as a postdoc. (I have retained detailed notes of that course and later seminars.) At times the class was intimidating, but it was mostly exhilarating. Dick then was assimilating q–series. He did much work on q–analogues and the Askey tableau, and—when Chip Morris joined—what developed into the Macdonald conjectures. George Andrews came, and we all read Hahn's 1949 paper on orthogonal polynomials and q–difference equations. It was an exciting time to be a student.

I soon discovered that Dick was very supportive of younger people. His encyclopedic knowledge of the literature simplified our struggles, and he offered quite a bit of mathematical advice, technical and otherwise. He supported our travel to conferences, and I sometimes stayed with him in the hotel. After graduating, I lived at his house in the summer! At these conferences I saw his much larger influence on the greater mathematics community, in shaping directions of research, connecting the right people, and proposing fundamental questions. For example, much of the subsequent work on Macdonald symmetric functions was initiated by his study of multidimensional q-beta integrals. Often Dick would tell a speaker an insightful way to prove his theorem, and have many references available. Sometimes Dick did not even know that area of mathematics! It is amazing, and I thank Dick for all of his support.

Here is an anecdote. Dick gave the following advice to Jim Wilson and me. When a senior person said something could be done, you should believe him, but when he said it could not be done, be skeptical. We took his advice when we asked him about the Askey tableau. The classical summation theorems were used as measures for orthogonal polynomials, so we wanted to know if the Pfaff–Saalschütz theorem could be used. He said no it would not work, but we did not believe him. Jim Wilson found the orthogonal rational functions that did work for that sum.

l to r: Dennis Stanton, Lynne Butler and Dick Askey
at the IMA in Minneapolis, Minnesota in 1988.

Dick Askey and Dennis Stanton

Dennis Stanton presenting at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013.

From: Sergeĭ Suslov (sergei@asu.edu)
Contributor: Sergeĭ Suslov: you always have been *American royalty* for me and my students

I first met Dick, Professor Askey, in front of the Kurchatov Institute in Moscow, when he visited the USSR in the mid 1980s. He energetically shook my hand and said "*Askey*". When we went to the seminar room for foreigners outside the institute, he immediately mentioned that "*the blackboard is too small and the (meeting) table is too long*". Dick was the first American I ever met. Later that evening he gave a talk at Gel'fand's seminar at Moscow State University, which unfortunately I missed because I couldn't get past the security guard in the main university building [sic]. After that there were numerous meetings/conversations/collaborations with Dick on different conferences all over the world: St. Petersburg, Florida, Hong Kong, Germany, to name a few, and later in the United States. I am very grateful to Dick and Liz for a wonderful possibility to stay in their home in Madison for a few days during my second visit to Canada and the United States (see pic below). Once again, for the first time, I had an opportunity to celebrate Thanksgiving holidays and enjoy the hospitality of the entire Askey family.

Sergeĭ Suslov, Dick Askey, Ramanujan's bust, and George Gasper
in Dick Askey's house in November 1993.

Most of Dick's professional life is related to the University of Wisconsin-Madison. It's well-known that Van Vleck Hall, the home of math department there, is named after the founder of modern theory of magnetism.[3] It's less known that he followed Dirac's ideas published in only one article without applications. Dirac visited Madison during his first world round

[3]Actually the building is named after American mathematician Edward Burr Van Vleck, the father of John Hasbrouck Van Vleck, the winner of the 1977 Nobel Prize in Physics.

trip and got paid $1800 for his stay in Madison, his first ever large royalty! Many years later (following the tradition?), Dick donated $1000 towards my first laptop computer, my father added $800 (enormous amount in the last years of the USSR!) and because of those two wonderful gifts I was able to LaTeX papers in Moscow in the following two years on my own laptop computer when I couldn't travel. Needless to say that the library of Dick's math department has practically every book on orthogonal polynomials and special functions. One of our joint papers with Dick and Mizan Rahman was written there in "regular office hours" and during the following evenings at Dick's house!

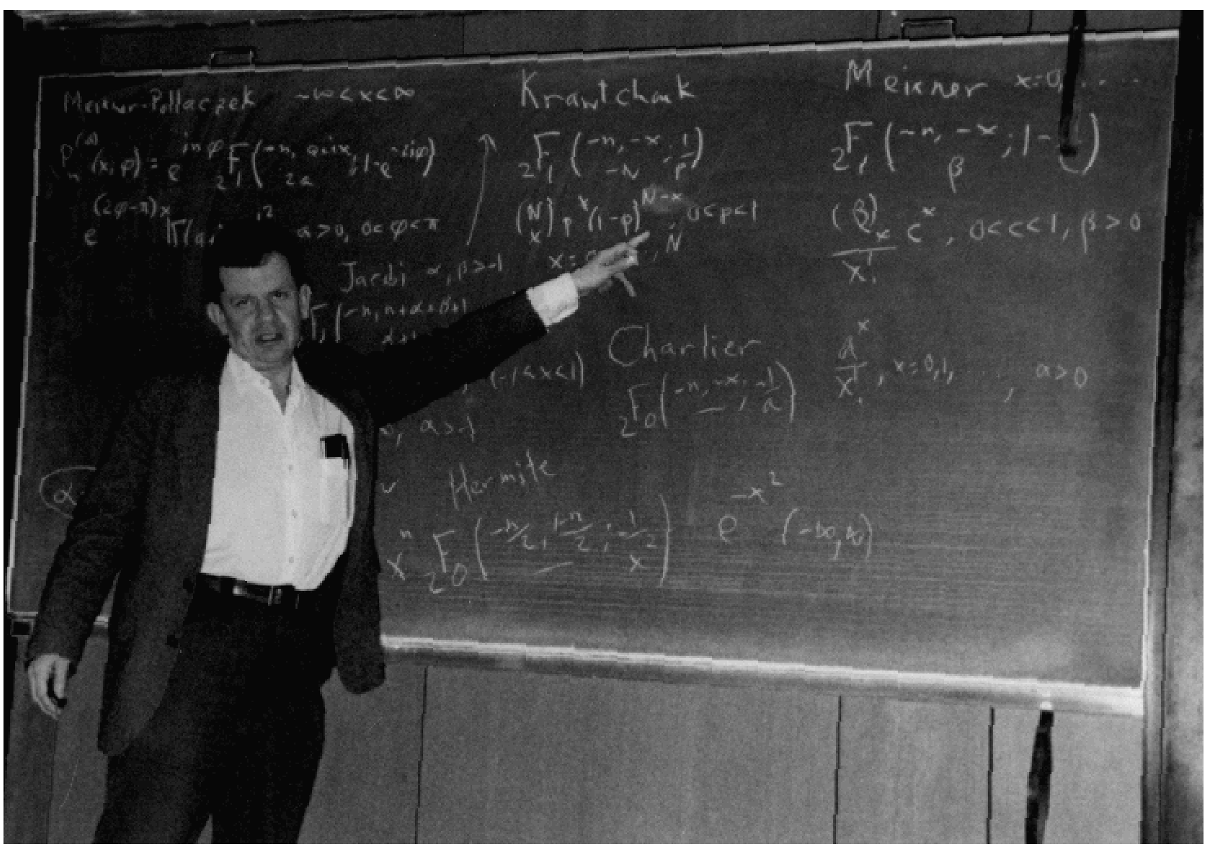
It is always exciting to discuss math (and physics) with Dick, and just bumping into him during Annual AMS Meetings. In Boston, for instance, I was lucky to stop him for quite a while in the covered path between hotels and the convention center in order to tell him that the first draft of our article S. I. Kryuchkov, S. K. Suslov and J. M. Vega-Guzman, *The minimum-uncertainty squeezed states for atoms and photons in a cavity*, had been rejected by J. Phys. B in 24 hours! Obviously, he found some words to encourage us and an extended revision was later published [86]. And this is just one example of Dick's friendly support of our research; he always has been an uncompromised judge of novelty and quality of publications in many areas of analysis and later in mathematical education.

When I was originally hired as a senior lecturer by the Arizona State University in the late 1990s, my first permanent job in the United States, Dick immediately said that in the classroom I would always be a professor! Next year, when my NSF grant application had been approved, he called the analysis program director in order to tell him that he had made the right choice! There are many other "real life stories" like that; for instance, Dick's presentation of our talk in the International Congress of Mathematicians in Warsaw, when Arnold Nikiforov "couldn't attend" in 1983; but this will be a very long story!

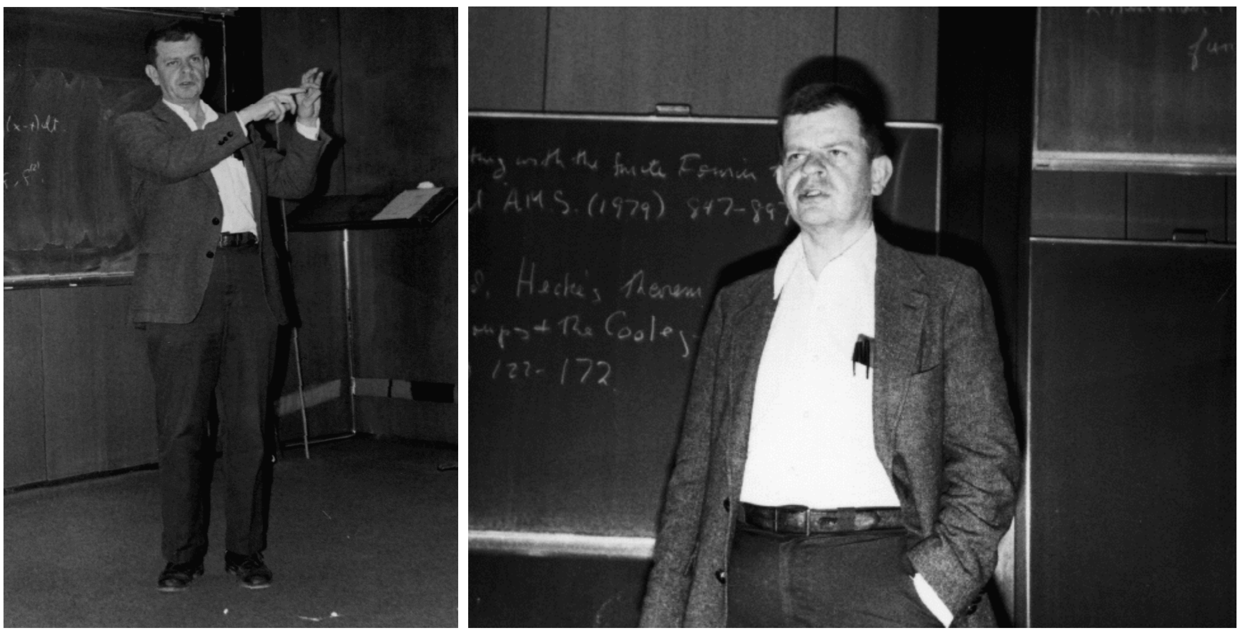
In conclusion, I believe that I may say that my English/American second language is too poor to express fully my respects, gratitude and friendship. We all love you Dick! You always had been an "American Royalty" for me and my students.

p.s. Dick's Erdős number is 2, and therefore my Erdős number is 3 (Erdős-Boas-Askey-Suslov, one of the paths) which is due to Dick. Dick's Landau number is 3 (Landau-Smorodinskiĭ-Suslov-Askey) as a result of our collaboration.

Dick and Liz Askey at Urbana, Illiois in 1987.



Dick Askey at Oberwolfach, Germany in March 1983



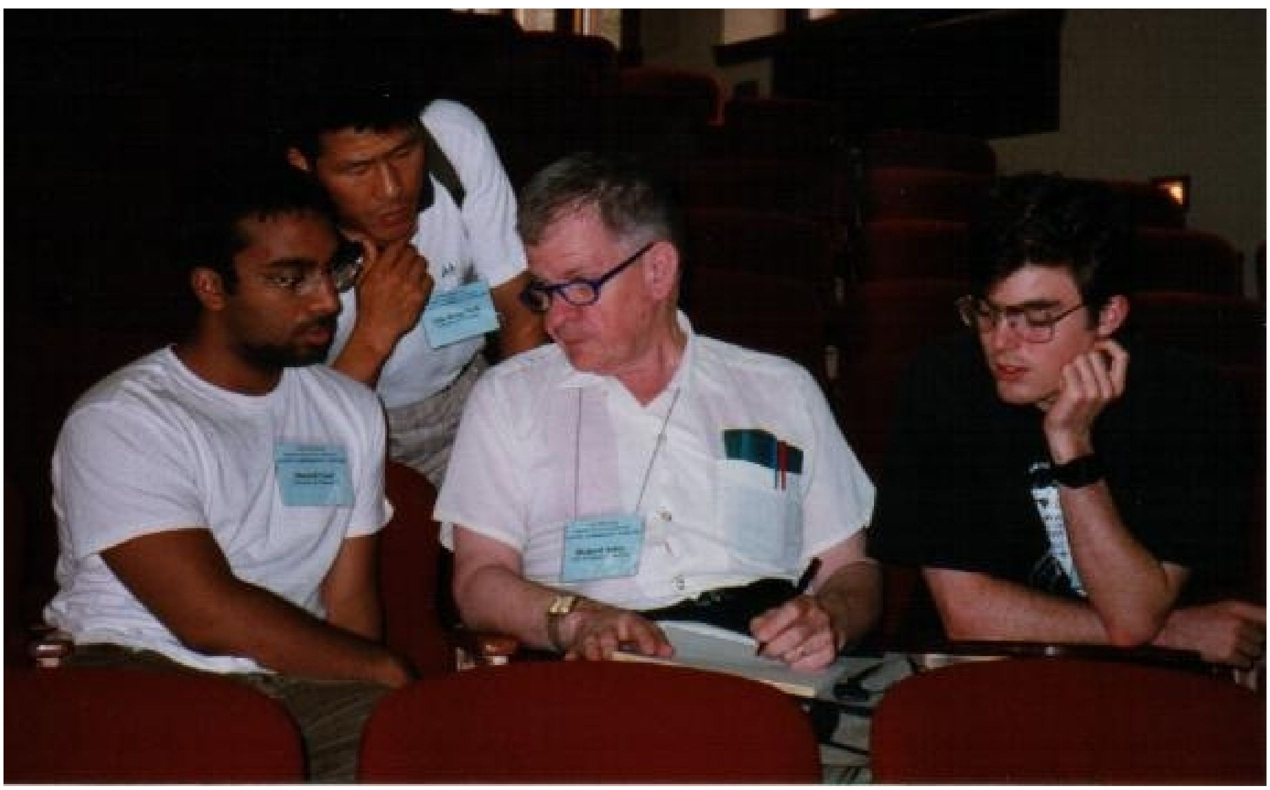
Dick Askey at Oberwolfach, Germany in March 1983

Conversations, conversations...

Conversations, conversations...

l to r: Doron Lubinsky, Paul Nevai, Dick Askey, Tom Koornwinder
attending OPSFA1 Polynômes Orthogonaux et Applications,
Bar-le-Duc, France, October 1984.

Dick Askey, Liz Askey and Mourad Ismail.



Dick Askey with graduate students.

l to r: Arun Ram, Dick Askey, Steve Milne, George Gasper, Shaun Cooper in 1994.

Dick Askey.

From: Nico Temme (Nico.Temme@cwi.nl)
Contributor: Nico Temme: Dick's enthusiasm was contagious

Dick Askey's sabbatical year in Amsterdam, 1969–1970

Nico Temme
CWI, Amsterdam

Dick visited the Mathematisch Centrum (MC) in Amsterdam[4] from July 1969–August 1970. He was guest researcher of the Department Toegepaste Wiskunde (TW, Applied Mathematics). Dick didn't know any of us, although he was familiar with the work of department chief Hans Lauwerier, who had written publications on asymptotic analysis and special functions. In fact these topics were extensively studied in the Netherlands by Prof. J. G. van der Corput, who became an expert in asymptotic problems in number theory. Van der Corput was one of the four founders of the MC in 1946 and he gave lectures on asymptotic analysis, he organized colloquia, and because of these activities many mathematicians in Amsterdam and in the country became interested in these topics. Among them was N. G. de Bruijn, who wrote the well-known book Asymptotic Methods in Analysis [58].

Around the time that Dick arrived in Amsterdam, there was some skepticism in the country about special functions, because computers could take over this topic. The special approach by Dick, in particular by linking elements of group theory with classical analysis, has caused his work to become more accepted and viewed from a different perspective.

Dick's enthusiasm was contagious, he was very active in writing publications during his sabbatical year, see his list at the end. The TW-reports became publications in journals. He gave many introductory lectures on orthogonal polynomials; eight lectures are bundled in the MC-report TC 51/70. At the end of that report Dick wrote: "*I would much rather want the reader to learn the above moral: do not study special functions for their own sakes. Without motivation and problems from some other field this area becomes sterile very fast. Of course this warning is not unique for special functions, but holds for any other specialized field of mathematics. And with this remark I close my series of lectures.*"

The lectures attracted always a large audience, because Dick managed to start with simple examples, and his mathematical approach was always very clear. Many people admired his capability of writing new formulas with one hand, and cleaning a used part of the black board with his other hand in one go. He didn't use notes, everything came right from memory.

Dick's sabbatical year was of great influence on a group of young MC-researchers: Tom Koornwinder, Herman Bavinck and I myself. We learned by visiting his lectures, from the problems and research topics suggested by Dick, from referee jobs he had to do and which he gave us to obtain our impression of the paper, and from daily contacts and discussions.

Among the advice he has given me, I remember the following two:

1. *When you take a formula from a book, be careful, it may not be correct.*
2. *When you do not remember a formula by heart, you do not understand it.*

[4]In 1983 the name of the institute changed into Centrum Wiskunde & Informatica (CWI).

Dick gave lectures in several colloquia outside the department:

1. Staff Colloquium of the Mathematical Institute of the University of Amsterdam: Applications of orthogonal polynomials, from projective spaces to numerical analysis.

2. MC-Colloquium Orthogonal Polynomials: 3 lectures.

3. MC Colloquium *Elementary topics explained from a higher point of view*: Certain rational functions whose power series have positive coefficients.

In the MC Annual Report 1970 we read in the Section *Approximation Theory and Special Functions*: Preparing a formula book in this area asked a lot of time. For that purpose, it turned out to be necessary to study extensively especially the discrete orthogonal polynomials (Charlier, Meixner and Hahn). Many new formulas were derived. One result found by Szegő on the positivity of the integral of a product of three Laguerre polynomials could be extended. About this and a number of similar results, an article will be published in collaboration with G. Gasper (Northwestern University, Illinois, USA).

MC-publications written by Dick:

1. MC Report TW 112, Orthogonal polynomials and positivity, 34 pages.

2. MC Report TW 113, Mean convergence of orthogonal series and Lagrange interpolation, 23 pages.

3. MC Report TW 114, Three notes on orthogonal polynomials, 17 pages.

4. MC-report TC 51/70, Eight lectures on orthogonal polynomials, 68 pages.

5. MC-Report ZW 1970-006, Certain rational functions whose power series have positive coefficients, 11 pages.

To download these publications, visit CWI's repository
https://ir.cwi.nl/#filter=author:Askey.

I am very thankful to Dick, for all educational contacts during his sabbatical year in Amsterdam, for the warm interest in my activities and for suggesting problems for my research on special functions and asymptotic analysis. For example, he was interested in the asymptotic properties of a class of polynomials biorthogonal on the unit circle. I have received many emails from colleagues who first asked Dick for help, and when there was an asymptotic element in the question, Dick referred to me. In this way, I have received many interesting research questions.

Nico Temme
Centrum Wiskunde & Informatica, Amsterdam, The Netherlands.

Photo taken during: "International Conference on Analysis, Applications and Computations, In Memory of Lee Lorch 1915-2014,"
September 28-30, 2015, The Fields Institute, Toronto, Canada.
l to r: Roderick Wong, Martin Muldoon, Vladimir Vinogradov, Chandler Davis, Robert Corless, Priscilla Greenwood, unknown, Mourad Ismail, Man-Duen Choi, Dick Askey, Man Wah Wong, Nico Temme.

Contribution #73 ——— Liber Amicorum ——— Dick Askey

From: Paul Terwilliger (terwilli@math.wisc.edu)
Contributor: Paul Terwilliger: their attitude about you approaches hero worship

Dear Dick,

This is your old friend Paul Terwilliger. When I emailed you in the past, I usually had some mathematical question or sought your advice. This time I will share some memories.

I remember the first time I met you, in 1984 or so. At that time I was a postdoc at Ohio State, and the event was an AMS meeting. I was scheduled to give a 10 minute talk on a newfangled object that I called a Leonard pair, and its relationship to the Leonard theorem about orthogonal polynomials. When I got up to speak, I looked out at the audience and noticed a man in the front row with a crew cut. After the talk he came up and introduced himself "*Hi, I'm Dick Askey*". At that moment, little did I know that you would play a major role in my subsequent career.

A year later I was hired as an assistant professor at U. Wisconsin, and I assume you had something to do with the hiring decision. Thank you for that, and your subsequent support over the years! I thrived at U. Wisconsin; at last count I wrote 117 papers and taught 41 graduate courses. The topics appear to be varied, but in most cases there is an intellectual thread going back to the Leonard theorem. That theorem and the surrounding ideas, proved to be a remarkable trove of mathematics. Around that topic I found mathematical gold everywhere I looked, and all I had to do was dig it up.

Needless to say, I was not the only person to see gold in that general mathematical area. A whole generation of people in special functions have spent their career studying the orthogonal polynomials in the Askey scheme. I know a lot of these people, and their attitude about you approaches hero worship. But your own attitude seemed remarkably humble to me. I remember you telling me about a conversation you had with the physicist Eugene Wigner. You told me that during the discussion Wigner always seemed to be a step ahead of you, and the whole conversation was profoundly humiliating. I had a hard time imagining you in that role, but found out later that Wigner was a Nobel prize winner. So I guess we will allow it. Your humble attitude toward Ramanujan is also notable in this regard.

Over the years I saw you always lifting up the people around you, and we all share a debt of gratitude. Bless You Dick!

Paul Terwilliger

Contribution #74 ——— Liber Amicorum ——— Dick Askey

From: Thotsaporn "Aek" Thanatipanonda (thotsaporn@gmail.com)
Contributor: "Aek" Thanatipanonda: I am sorry that I did not say hi to you at JMM

It was back in September 2000, when I took an undergraduate combinatorics class with Prof. Richard Askey. It was my first mathematics class that was taught by a real mathematician. To my surprise, although at the time Prof. Askey was almost 70 years old, his enthusiasm was through the roof and you can feel that during the lectures. After taking his class, I decided to continue my study in Mathematics. He also helped me published the result from class project on bijective proof between number of inversions and major indexes on the permutation of length n. Without his letter of recommendation, it would not be possible to attend a graduate school like Rutgers University. I know I owe him a lot. I always feel very lucky that I was able to get a chance to meet and study with Prof. Askey. It is a wonderful feeling to have support from such a giant in the field. His motivation and inspiration will always be with me forever. Lastly I am sorry that I did not say hi to him when we met at JMM back in 2017 and 2018. I am too shy.

Sincerely,
Thotsaporn "Aek" Thanatipanonda

Contribution #75 ——— Liber Amicorum ——— Dick Askey

From: Walter Van Assche (walter.vanassche@kuleuven.be)
Contributor: Walter Van Assche: thanks Dick for your valuable advice

Obviously Dick Askey was the world expert on special functions when I started my PhD research in the early 1980's. I had secured a travel grant from the Belgian government allowing me to travel and visit a research laboratory abroad and I needed to find an appropriate place to visit. My PhD advisor suggested that I write Askey in Madison, Wisconsin for advice and I took pen and paper (no e-mail those days) and wrote Prof. Richard Askey a letter.

To my surprise, he wrote back right away, and a week later I received his kind letter. He suggested two possible places in Europe: the first was to visit J.S. Dehesa in Granada, Spain, who had written a paper with Paul Nevai on the asymptotic distribution of zeros of orthogonal polynomials in SIAM J. Math. Anal. (1979) [99]. An excellent choice: both Dehesa and Nevai later became good friends with whom I wrote several good papers. However, I went for Dick's second suggestion: to visit Daniel Bessis at Centre d'Études Nucléaires de Saclay, near Paris. Dick said that he and his collaborators had done interesting work on orthogonal polynomials on Julia sets related to the iteration of a polynomial [50], and Jeff Geronimo was visiting them in 1984. So on Dick's advice, I visited Saclay in 1984 and I became very good friends with Jeff Geronimo.

So thanks Dick for your valuable advice which resulted not only in a very good start of my career, but also in long lasting friendships with experts in orthogonal polynomials. Naturally I met Dick later at several conferences and I enjoyed talking with him about orthogonal polynomials, special functions, q-series and mathematics education.

From: Luc Vinet (luc.vinet@yahoo.com)
Contributor: Luc Vinet: a modest tribute to Dick Askey, a friendly giant

A MODEST TRIBUTE TO DICK ASKEY, A FRIENDLY GIANT

Luc Vinet, Centre de recherches mathématiques, Université de Montréal, Canada.

I have not known Dick very well nor for a very long time but through my encounters with him over the last years, I have developed a sincere fondness for him and it felt as this was reciprocal. Also, I have been acquainted with Dick's mathematical achievements for the past thirty years only but it was like a revelation when it happened.

Looking for exact solutions to interesting physical models is a well-proven tradition in mathematical physics to which I belong. Special functions abound in this context. There is also a belief that models can be solved exactly because they possess symmetries. Looking for mathematical structures expressing symmetries and their representations in terms of special functions is often the key to making advances in physical understanding and discovering solvable systems. Hence my interest in exploring the symmetries encoded in orthogonal polynomials and vice-versa.

Quantum groups were being actively developed around the time I was on sabbatical at UCLA in 1989. Uninformed of the immense work on q-series, quite naively, I had the feeling that interesting functions should emerge from the representations of these quantum groups. Weather being always nice in LA, the campus bookstore would often put books on display outside. One lucky morning, as I was walking to the Physics Department, I stumbled by chance on the book of Gasper and Rahman prefaced by Dick. It was the light I was looking for—an amazing eye-opener—and how I learned about the Askey-Wilson polynomials and caught the q-disease.

This got Floreanini and I started on the connection between quantum groups and algebras and q-special functions. Later, with Lapointe I worked on the Jack and Macdonald polynomials providing their raising operators solving Macdonald conjectures. In so doing, and at about the same time realizing that we were independently pursuing parallel research on quadratic algebras and superintegrable systems, Zhedanov and I joined forces and began our on-going collaboration. I did not meet Dick much during those years, moving I suppose in different circles. Moreover from 1999 to 2010, I got rather busy with university administration.

From 2010 onward, after I had finished my terms as Provost and Rector, I think I met Dick almost every year. I apologize for having spoken too much about me but this aimed to stage my few personal anecdotes about Dick. They are all rooted in various conversations and meals that we enjoyed together.

In spite of his encyclopedic knowledge, I do not think that Algebra was Dick's area of predilection. Often times, he came to me saying: *Please stop talking about the Askey-Wilson algebra, I have nothing to do with that—call it the Zhedanov algebra.* This gives an example of his modesty. Alas Dick, it is too late; I am afraid the name will stick and that you wont be able to get rid of your algebra!

I wish to mention a question that Dick has regularly put to me and that has yet to be resolved. Seven or eight years ago, Zhedanov and I sorted out properties of the Bannai- Ito polynomials and began talking about -1 orthogonal polynomials. Dick kept saying: you ought to be able to give similar treatments for other roots of unity. There should be connections to sieved polynomials. Go ask Mourad. Well we asked Mourad and wrote with him papers ... but on different topics. I keep remembering Dick's words and given his legendary insight, I am sure there is much to be understood there. I hope to make good some day on this inspiration that Dick is still instilling directly. Anyone wishing to join is welcome.

Dick is also passionate about mathematical education. Here again my discussions with him on that topic are always on my mind as I try to increase the role of the CRM in this sector in Quebec.

Last, I want to stress again Dick's kindness. Two years ago, Paul Terwilliger invited me to give a seminar in Madison. Unexpectedly, Dick came. This really touched me because it was especially difficult for him as he was in the process of moving.

Dick, many many thanks for your mathematics and for encouraging us all to climb on the shoulders of the giant that you are.

Most sincerely,
Luc

Contribution #77 ——— Liber Amicorum ——— Dick Askey

From: Hans Volkmer (volkmer@uwm.edu)
Contributor: Hans Volkmer: one of the most beautiful and useful areas in mathematics

One of my teachers at the University of Konstanz (Germany) was Friedrich Wilhelm Schäfke who jointly with Josef Meixner wrote the book "Mathieusche Funktionen und Sphäroidfunktionen" in 1954 [96]. Josef Meixner is known in the theory of orthogonal polynomials through his introduction of the Meixner polynomials. Under the supervision of Schäfke, I wrote my dissertation in the area of Special Functions. Later, I continued research in Special Functions, although mostly in the "land beyond Bessel" as Felix Arscott used to say [16].

Dick Askey is one of the editors of the DLMF [59] project, to which I contributed the chapters on Lamé and spheroidal functions. If I remember correctly, Dick Askey was a letter writer for one of my promotion cases. I am very grateful for this. Of course, when you work in the area of Special Functions, you frequently come across results of Dick Askey.

In January 2000, Dick Askey presented the talk "Some inequalities suggested by asymptotics" at the *Symposium on Asymptotics* in San Diego, California. In this talk Dick made some conjectures on the positivity of kernels related to the Cesàro means for expansions in Gegenbauer polynomials. This led to my paper "On some conjectures of Askey" published in the journal Asymptotic Analysis in 2001 [129].

I am very thankful for Dick Askey's contributions to the area of Special Functions which is certainly one of the most beautiful and useful areas in mathematics.

Hans Volkmer

From: Ole Warnaar (o.warnaar@maths.uq.edu.au) and Wadim Zudilin (wzudilin@gmail.com)
Contributor: Ole Warnaar and Wadim Zudilin: q-rious and q-riouser

q-rious and q-riouser

S. Ole Warnaar and Wadim Zudilin
To our friend Dick Askey
September 2019

Dick Askey is known not just for his beautiful mathematics and his many amazing theorems, but also for posing numerous interesting and important open problems. Dick being Dick, these problems are hardly ever isolated, and often intended to demonstrate the unity of analysis, number theory and combinatorics. On this occasion we wish to take the reader down the rabbit hole created by one such problem, published as *Advanced Problem 6514* by the American Mathematical Monthly in April 1986 [34, 36]. Dick's inspiration for the problem was derived from the Macdonald–Morris constant term conjecture for the root system G_2 [93, 98] as well as much earlier work of P. Chebyshev [126] and E. Landau [88, p. 116] on the integrality of factorial ratios. Problem 6514 asks for a proof of the integrality of

$$A(m,n)=\frac{(3m+3n)!\,(3n)!\,(2m)!\,(2n)!}{(2m+3n)!\,(m+2n)!\,(m+n)!\,m!\,n!\,n!}$$

for all non-negative integers m and n.

There are multiple reasons—some of them very deep, see e.g., [54, 109, 117, 118]—for wanting to classify integer-valued factorial ratios such as Chebyshev's

$$C(n)=\frac{(30n)!\,n!}{(15n)!\,(10n)!\,(6n)!}\,.$$

Given a particular such ratio, integrality can always be verified by computing the p-adic order of the factorials entering the quotient. This is exactly what all eight solvers of Problem 6514 did. Such a verification, however, provides little insight into which ratios are integral and which ones are not, and from the editorial comments to the problem it is clear that Dick would have liked to see other types of solutions too. Indeed, it is remarked that

> *[ ] the editor [read: Dick Askey] feels there is still room for other methods, involving perhaps combinatorial interpretations or manipulation of generating functions. In this particular case, the proposer remarks that $A(m,n)$ should be the constant term of the Laurent polynomial*

$$\begin{aligned}&\big((1-x)(1-1/x)(1-y)(1-1/y)(1-y/x)(1-x/y)\big)^m\\&\qquad\times\big((1-xy)(1-1/xy)(1-y/x^2)(1-x^2/y)(1-y^2/x)(1-x/y^2)\big)^n.\end{aligned}$$

Incidentally, L. Habsieger [79] and D. Zeilberger [133] both proved the G_2 Macdonald–Morris constant term conjecture shortly after Dick Askey posed his problem. The submission dates of their respective papers (the 12$^{\text{th}}$ of May and the 2$^{\text{nd}}$ of June 1986) were well within the deadline of the 31$^{\text{st}}$ of August for submitting solutions to Problem 6514 to the Monthly. In fact, in the acknowledgment of his paper Zeilberger thanks Dick Askey for “rekindling his interest in the Macdonald conjecture”, so maybe he should belatedly be considered the 9$^{\text{th}}$ solver of Askey's problem.

The height of a factorial ratio is the number of factorials in the denominator minus the number of factorials in the numerator, so that the height of $A(m,n)$ is two whereas the height of $C(n)$ is one. A one-parameter family of height-k factorial ratios

$$F(n)=\frac{(a_1\,n)!\cdots(a_\ell\,n)!}{(b_1\,n)!\cdots(b_{k+\ell}\,n)!}$$

is balanced if $a_1+\dots+a_\ell=b_1+\dots+b_{k+\ell}$. All balanced, integral, height-one factorial ratios $F(n)$ were classified in 2009 by J. Bober [54]. In relation to this classification we should mention F. Rodriguez-Villegas' observation [109] that if $F(n)$ is a balanced, height-one factorial ratio then the hypergeometric function $\sum_{n\geq 0}F(n)z^n$ is algebraic if and only if $F(n)$ is integral. This observation was key to Bober's proof, allowing him to use the Beukers-Heckman classification [51] of ${}_nF_{n-1}$ hypergeometric functions with finite monodromy group. A proof not reliant on the Beukers-Heckman theory was recently found by K. Soundararajan [117]. By extending his method he also obtained a partial classification in the height-two case [118].

Despite the availability of the number-theoretic, p-adic approach to factorial ratios, the question of integrality is very interesting from a purely combinatorial point of view. The simplest example is of course provided by the height-one binomial coefficients

$$\frac{(m+n)!}{m!\,n!},$$

whose integrality can be established combinatorially (as well as probabilistically, algebraically, etc.) with little effort. However, to the best of our knowledge, no combinatorial proof is known of the integrality of Chebyshev's $C(n)$.

A related open problem arises from our joint work [131] from 2011. In [131] we observed that if each factorial $m!$ in an integral factorial ratio is replaced by a q-factorial

$$[m]!=[m]_q!=\prod_{i=1}^{m}\frac{1-q^i}{1-q},$$

then the resulting q-factorial ratio is a polynomial with non-negative integer coefficients. The polynomiality and integrality parts are trivial but the positivity—which was referred to in [131] as 'q-rious positivity'—is completely open. The only (irreducible) cases that are proven are the three two-parameter families of height one given by

$$\frac{[m+n]!}{[m]!\,[n]!},\qquad \frac{[2m]!\,[2n]!}{[m]!\,[n]!\,[m+n]},\qquad \frac{[m]!\,[2n]!}{[2m]!\,[n]!\,[n-m]!}\quad (m\leq n),$$

where the first family corresponds to the q-binomial coefficients and the second family to the q-super Catalan numbers. In the q-case no arithmetic approach is available, and given the lack of combinatorial methods to deal with integrality, a combinatorial approach to q-rious positivity seems hopeless.[5] Perhaps the most tractable problem is to analytically prove, along the lines of [131], the positivity of the known two-parameter families of height two, such as

$$A_q(m,n)=\frac{[3m+3n]!\,[3n]!\,[2m]!\,[2n]!}{[2m+3n]!\,[m+2n]!\,[m+n]!\,[m]!\,[n]!\,[n]!}\in\mathbb{Z}[q]$$

[5] There are of course countless methods to show that the q-binomial coefficients have non-negative integer coefficients, but no combinatorial interpretation of the q-super Catalan numbers is known. In fact, not even a combinatorial interpretation of the ordinary super Catalan numbers is known.

and

$$C_q(m,n) = \frac{[6m+30n]!\,[n]!}{[3m+15n]!\,[2m+10n]!\,[m]!\,[6n]!} \in \mathbb{Z}[q].$$

For the first family, which is the q-analogue of $A(m,n)$, it is known that [56, 79, 133]

$$A_q(m,n) = \operatorname*{CT}_{x,y}\Big[(x,q/x,y,q/y,y/x,qx/y;q)_m (xy,q/xy,y/x^2,qx^2/y,y^2/x,qx/y^2;q)_n\Big],$$

where $(a_1,\dots,a_k;q)_n := \prod_{i=1}^{k}\prod_{j=1}^{n}(1-a_i q^{j-1})$. This interpretation as a G_2 constant term gives little insight into the positivity of the coefficients. It would appear that the second two-parameter family has not occurred before. For $q=1$ it arose earlier this year in the (partial) classification of height-two factorial ratios by Soundararajan [118] mentioned above. It should be noted that if one were to prove the q-rious positivity of $C_q(m,n)$ then this immediately would imply the positivity of the q-analogue of Chebyshev's factorial ratio since

$$C_q(n) = \frac{[30n]!\,[n]!}{[15n]!\,[10n]!\,[6n]!} = C_q(0,n).$$

Ole Warnaar and Dick Askey at the *Askey 80th Birthday Conference*,
Madison, Wisconsin, USA in December 2013.

Contribution #79 ——— Liber Amicorum ——— Dick Askey

From: Stephen Wainger (wainger@math.wisc.edu)
Contributor: Stephen Wainger: the field of mathematics owes Dick a huge debt

The field of Mathematics owes Dick a huge debt for restarting an exciting area of mathematics, special functions. And I personally want to express to Dick my appreciation first of all for introducing me to the area of special functions. Then I want to express my appreciation for our collaboration on eight papers. I also owe Dick thanks for arranging for my job at the University of Wisconsin. I wish Dick a speedy recovery so we can collaborate on one more paper.

From: Patsy Wang-Iverson (pwangiverson@gmail.com)
Contributor: Patsy Wang-Iverson: you may find each other interesting

Dick and I began communicating online in the late 90's over our common interest in the TIMSS (then called the Third International Mathematics & Science Study, with the T subsequently changed to Trends with the 2003 study) data and the Singapore approach to teaching mathematics. We subsequently obtained a small NSF grant (after he introduced me to Susan Sclafani, then Assistant Secretary of Education) that allowed us to convene a TIMSS Work Session at Wingspread in 2004, bringing together a small group of mathematicians and mathematics educators to analyze the TIMSS videos. The results were organized by Cathy Kessel and uploaded to the Research for Better Schools website, with Dick and me as Co-editors (no longer available).

Dick opened many doors for me, including introducing me to Madge Goldman, President of the Gabriella & Paul Rosenbaum Foundation, saying, "*You may find each other interesting.*" We three then embarked upon an effort to study use of Singapore Mathematics in American classrooms. Dick played a vital role as a sounding board and critical reviewer for Madge, examining carefully materials she sent him before publication.

Dick and I collaborated on a number of presentations at national and state conferences for mathematics educators and got together at national and international conferences (see photo from *12th International Conference on Mathematical Education* (ICME-12) in Seoul, South Korea).

We transcended the work relationship to become good friends. I had the opportunity to visit him and Liz in Madison when he called to tell me I might be interested in attending the 2008 MathFest in Madison, as Erik Demaine was to be the keynote lecturer. After the conference, Dick and Liz took David and me sightseeing to visit a Frank Lloyd Wright house, and, of course, the famous cheese store. When we went to the house, and I commented on the walls of books throughout the house, Dick said he was in the process of trying to downsize his collections. Not doing so, he said, would be akin to child abuse.

In addition to mathematics education, Dick, Liz, a children's librarian, and I shared a love for children's books, in particular pop-up books. I treasure the Alice in Wonderland pop-up birthday card they sent me.

I called Dick on June 4 to wish him a Happy Birthday. He sounded good, thus it was a shock to receive the news about his deteriorating health from Suzanne.

I hope to see you in the near future, Dick.

With love,
Patsy Wang-Iverson

Dick Askey on a seesaw in 2012.

Contribution #81 ——— Liber Amicorum ——— Dick Askey

From: Lauren Williams (williams@math.harvard.edu)
Contributor: Lauren Williams: beautiful mathematics

Dear Dick,

Thank you for your beautiful mathematics; it has been truly inspirational to me. In particular, your remarkable Askey-Wilson polynomials, and their connection to the asymmetric simple exclusion process (ASEP)—a Markov chain which was introduced as a model for translation in protein synthesis—have been a key player in my work over the past decade. Sylvie Corteel and I introduced staircase tableaux to give a combinatorial model for the stationary distribution of the ASEP and for the moments of Askey-Wilson polynomials. But there are things that we don't yet understand, and I'm looking forward to puzzling out more secrets of Askey-Wilson polynomials in the years to come.

With gratitude,
Lauren

Contribution #82 ——— Liber Amicorum ——— Dick Askey

From: Roderick Wong (rscwong@gmail.com)
Contributor: Roderick S. C. Wong: a great leader in our field of special functions

Dick Askey is undoubtedly a great leader in our field of Special Functions, and many of you have already expressed this recognition. Here I wish to share with you a personal experience which has convinced me of his leadership position.

In 1968 (maybe '69), I met Arthur Erdélyi in Edmonton, where I was a PhD student and just about to graduate. Since Max Wyman (my supervisor) was a good friend of Erdélyi, he suggested I see Erdélyi and seek for advice. To my surprise, Erdélyi told me that the fields of Asymptotics and Special Functions were dead, and advised me to find something else to work on. As you can imagine, I was very depressed by this information. Fortunately, on a separate occasion, also in 1968 (or 69), I ran into Dick Askey in the Department of Mathematics in Edmonton. He gave me a quite different impression.

The occasion on which I ran into Dick in Edmonton was itself interesting. One day early in the morning at around 5:00 or 5:30 am, I saw Dick wandering alone in the Math Dept. I asked him how come he was in the Department so early in the morning. He said he arrived in Edmonton at around 10:00 pm on the previous day, and found that the day was still bright. Then he woke up at around 3:00 am and the sun was already up. Since no one else was around, I had a chance to carry on a long talk with Dick.

In recent years, I have developed an asymptotic theory for 2$^{\text{nd}}$ -order linear difference equations. For years I have been asking myself who had suggested me to work on this problem. I know for sure it is not Max Wyman nor Frank Olver, since they were not that interested in this topic. It's only recently I came to the conclusion that this suggestion must have come from Dick Askey, only he would know the importance of asymptotics for linear recurrences.

Over the the past 50 years, we have witnessed how Dick built up a group of excellent young mathematicians working around him, and how he revitalised the field of Special Functions.

Dick Askey at City University of Hong Kong.

To conclude, I will quote a statement made by G.-C. Rota: "*It is hard to tell who will win the fraternal joust between the abstract but penetrating concepts of the theory of group representations and the concrete but permanent formulas of the theory of special functions. We place our bets on the latter.*" *Indiscrete Thoughts* (1997) [112, p. 231].

Dick, stay strong. We need you to be around.

Dick Askey and Roderick Wong at the *Askey 80^{th} Birthday Conference*, Madison, Wisconsin, USA in December 2013. The picture was taken by Patsy Wang-Iverson.

From: Hung-Hsi Wu (wu@berkeley.edu)
Contributor: Hung-Hsi Wu: accidentally dragged into the Math Wars in 1992

I first met Dick in a San Francisco Hotel during the Joint Meeting in January of 1995. One of the sessions there was devoted to the school math education reform. The Math Wars were beginning to rage nationwide in 1995, and I believe that session was one of the reasons that brought Dick to San Francisco. He and I had been corresponding by email about school math education since 1992, so that by 1995, we were already comfortable sharing our views on the subject. I first approached him when I was accidentally dragged into the Math Wars in 1992 and I needed any ally I could get. I happened to remember that Dick had written about education in the AMS Notices not long before that, so I mailed him my first education article. He told me many years later that he was actually very suspicious of the article when he first got it, because he had seen too many mathematicians mouth off about education. I suppose he was relieved that I was not one of them, and that occasion marked the beginning of a friendship that has lasted twenty-five years. By year 2000 I was already doing education full-time, but Dick kept up his research work and our communications were sporadic. Nevertheless, we slowly found out that our outlooks in life were not that different, and our philosophical stances in mathematics were also similar—the disparity in our respective fields notwithstanding. So we had lots of things to talk about. By the time I retired in 2009, he had been retired for a few years already and our phone conversations became more frequent. Until his latest illness, we probably called each other daily for the past three or four years, and our favorite topics were education, mathematics, and politics (not necessarily in this order). I think my life in education would have been intolerable but for Dick's sympathetic backing and good advice. He knows how badly I want our daily conversations to resume!

From: Doron Zeilberger (doronzeil@gmail.com)
Contributor: Doron Zeilberger: a special (and **very important**!) guru indeed

Dick Askey: a special (and very important!) guru indeed
Doron Zeilberger

Special Functions are functions that occur so often that they deserve a name, but even more important than functions are *people*, and Dick Askey is one of the most special people I have ever met. In this brief note, let me mention a small sample of the very significant mathematics that he inspired.

- It was Dick who challenged Dominique Foata to find a combinatorial proof of the Mehler formula, and that lead to Foata's beautiful combinatorial approach to Special Functions, pursued by him, his collaborators, and many others.
- It was Dick who had brilliant PhD students, including Dennis Stanton, that became the absolute unit of quality for all PhD students. Being x–Dennis with $x > \frac{1}{2}$ means that the student is very good. Of course $x \leq 1$, and the inequality is *sharp*, with only one case (guess who?) achieving $x = 1$.
- Dick is very passionate about mathematics education, something very rate amongst research mathematicians, and is a 'gadfly' (with co-gadfly George Andrews) about what they deemed misguided reforms in K–12 education, stating that *Good intentions are not enough*. Dick did not just criticize, but set an example by writing insightful articles for the *Mathematics Teacher*, the leading periodical for high-school teachers, about Fibonacci and Lucas numbers.
- The 'Askey scheme' (aka *tableau d'Askey*), hanging in my office (nicely designed by Jacques Labelle), was the **conceptual skeleton** of lots of great research in special functions, highlighting the Askey hypergeometric hierarchy, with the Racah and Askey–Wilson polynomials on the top.
- Dick caught the q–disease, and along with George Andrews and others (Mizan Rahman, George Gasper, Dennis Stanton, Frank Garvan to name a few) inspired lots of insightful q-*analogs* of classical theorems, that led to insightful combinatorial interpretations (Xavier Viennot and his school, Bill Chen and his school, and many others).
- The *Askey-Gasper inequality* was the **crucial fact** needed in the proof of one of the most *important* open problems of the 20$^{\text{th}}$ century, Louis de Branges' proof of the Bieberbach conjecture.

  Finally, let me mention Dick's influence on myself.

- Dick was a great professional father-figure. He is the one who challenged me to prove George Andrews' q–Dyson conjecture, that I did, in 1983 (published in 1985) in collaboration with Dave Bressoud. He is the one who challenged me to prove the G_2 case of Ian Macdonald's Constant Term Conjecture (that I did, also done independently by Laurent Habsieger), where I used the Dixon identity, that I learned from him and from Foata. Dick also challenged me to prove the G_2–dual case of the same conjecture, that I did using what I called the *Stembridge-Stanton* trick. That method was later used by another whiz, Frank Garvan, to prove the F_4 case.
- Dick was an implicit, but very strong, influence in the development of so-called: *Wilf-Zeilberger algorithmic proof theory*.

Thank you Dick!

From: Alexei Zhedanov (zhedanov@yahoo.com)
Contributor: Alexei Zhedanov: I was born under the *constellation of Askey*

As a mathematician, I was born under the *constellation of Askey*.

Salieri, the main character of Pushkin's drama "Mozart and Salieri" «Мóцарт и Сальéри» says (Translated by Alan Shaw):

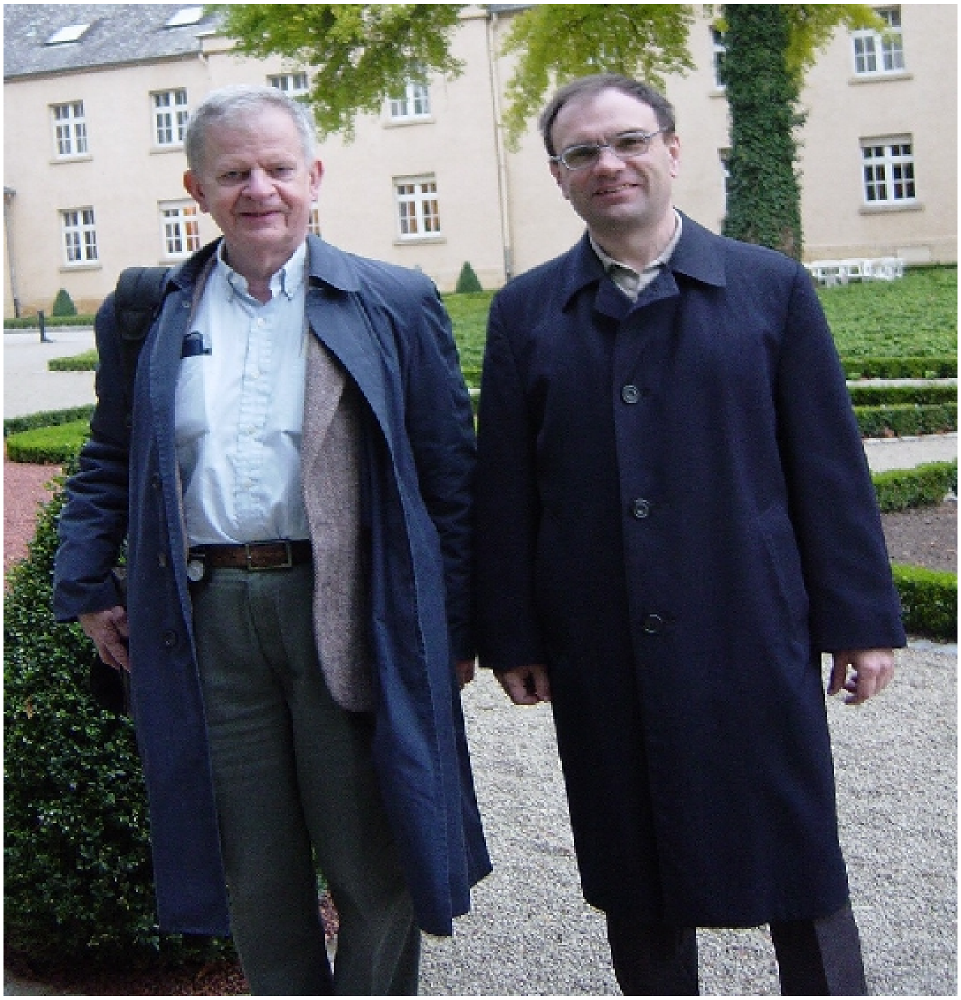
Что говорю? Когда великий Глюк	*What am I saying? When great Gluck himself*
Явился и открыл нам новы тайны	*Appeared, unfolding us new mysteries*
(Глубокие, пленительные тайны),	*(And deep enthralling mysteries they were),*
Не бросил ли я все, что прежде знал,	*Did I not give up all I'd known before,*
Что так любил, чему так жарко верил,	*And dearly loved and fervently believed in?*
И не пошел ли бодро вслед за ним	*Did I not briskly follow him, without*
Безропотно, как тот, кто заблуждался	*A murmur, like a man who's lost his way,*
И встречным послан в сторону иную?	*And meets another who can set him right?*

Askey has played the similar role of the "great Gluck". The area of my research changed when Askey–Wilson polynomials appeared in the mathematical World.

During several years, I was trying to find a simple algebraic explanation of the Askey–Wilson polynomials. This was finally done in 1991 when the Askey–Wilson algebra $AW(3)$ was shown to describe all basic properties of these remarkable polynomials: duality, recurrence relation and q-difference equation. Although Dick always warned against the name "Askey–Wilson algebra", I still believe that this name is quite appropriate, stressing the main role of the objects to which the algebra is applied.

In the following years, my activity was mainly related to Askey's topics. Together with my friends and colleagues (I can mention those with whom I collaborated for many years: Slava Spiridonov, Satoshi Tsujimoto and Luc Vinet), we have tried to generalize and to extend the Askey scheme in different directions: Bannai–Ito polynomials, elliptic biorthogonal rational functions, algebraic Heun operators, etc.

I would like to say many thanks to Dick for his influence to my life and to my results.



l to r: Dick Askey, Alexei Zhedanov at Difference Equations and Special Functions, Bexbach, Saarland, Germany in October 26–30, 2002.

l to r: Kathy Driver, Dick Askey, Alexei Zhedanov at *Conference on Difference Equations, Special Functions and Applications*, Munich, Germany in July 2005.

Bibliography

[1] W. Abikoff, J. S. Birman, and K. Kuiken, editors. *The mathematical legacy of Wilhelm Magnus: groups, geometry and special functions*, volume 169 of *Contemporary Mathematics*. American Mathematical Society, Providence, RI, 1994.

[2] M. Abramowitz and I. A. Stegun. *Handbook of mathematical functions with formulas, graphs, and mathematical tables*, volume 55 of *National Bureau of Standards Applied Mathematics Series*. U.S. Government Printing Office, Washington, D.C., 1972.

[3] W. A. Al-Salam and T. S. Chihara. Convolutions of orthonormal polynomials. *SIAM Journal on Mathematical Analysis*, 7(1):16–28, 1976.

[4] K. Alladi. *Ramanujan's place in the world of mathematics*. Springer, New Delhi, 2013. Essays providing a comparative study, With a foreword by George E. Andrews.

[5] H. Alzer and M. K. Kwong. A refinement of Vietoris' inequality for cosine polynomials. *Analysis and Applications*, 14(5):615–629, 2016.

[6] H. Alzer and M. K. Kwong. On a trigonometric inequality of Askey and Steinig. *Asymptotic Analysis*, 106(3-4):233–249, 2018.

[7] H. Alzer and M. K. Kwong. On two trigonometric inequalities of Askey and Steinig. *New York Journal of Mathematics*, 26:28–36, 2020.

[8] G. Andrews, G. Gasper, M. Ismail, and P. Nevai. Dedication. *SIAM Journal on Mathematical Analysis*, 25(2):vii–ix, 1994.

[9] G. E. Andrews. Problems and prospects for basic hypergeometric functions. In *Theory and application of special functions (Proc. Advanced Sem., Math. Res. Center, Univ. Wisconsin, Madison, Wis., 1975)*, pages 191–224. Math. Res. Center, Univ. Wisconsin, Publ. No. 35, 1975.

[10] G. E. Andrews. The fifth and seventh order mock theta functions. *Transactions of the American Mathematical Society*, 293(1):113–134, 1986.

[11] G. E. Andrews. *q-series: their development and application in analysis, number theory, combinatorics, physics, and computer algebra*, volume 66 of *CBMS Regional Conference Series in Mathematics*. Published for the Conference Board of the Mathematical Sciences, Washington, DC; by the American Mathematical Society, Providence, RI, 1986.

[12] G. E. Andrews. *Number theory*. Dover Publications, Inc., New York, 1994. Corrected reprint of the 1971 original [Dover, New York; MR0309838 (46 #8943)].

[13] G. E. Andrews. On a conjecture of Peter Borwein. *Journal of Symbolic Computation*, 20(5-6):487–501, 1995. Symbolic computation in combinatorics Δ_1 (Ithaca, NY, 1993).

[14] G. E. Andrews, R. Askey, and R. Roy. *Special functions*, volume 71 of *Encyclopedia of Mathematics and its Applications*. Cambridge University Press, Cambridge, 1999.

[15] G. E. Andrews, R. A. Askey, B. C. Berndt, K. G. Ramanathan, and R. A. Rankin, editors. *Ramanujan revisited*. Academic Press, Inc., Boston, MA, 1988.

[16] F. M. Arscott. The land beyond Bessel: a survey of higher special functions. In *Ordinary and partial differential equations (Proc. Sixth Conf., Univ. Dundee, Dundee, 1980)*, volume 846 of *Lecture Notes in Math.*, pages 26–45. Springer, Berlin, 1981.

[17] R. Askey. Mean convergence of orthogonal series and Lagrange interpolation. *Math. Centrum Amsterdam Afd. Toegepaste Wisk. Rep. TW*, 113:23, 1969.

[18] R. Askey. Mean convergence of orthogonal series and Lagrange interpolation. *Acta Mathematica. Academiae Scientiarum Hungaricae*, 23:71–85, 1972.

[19] R. Askey. *Orthogonal polynomials and special functions*. Society for Industrial and Applied Mathematics, Philadelphia, Pa., 1975.

[20] R. Askey. An elementary evaluation of a beta type integral. *Indian Journal of Pure and Applied Mathematics*, 14(7):892–895, 1983.

[21] R. Askey. Orthogonal polynomials and theta functions. In *Theta functions—Bowdoin 1987, Part 2 (Brunswick, ME, 1987)*, volume 49 of *Proc. Sympos. Pure Math.*, pages 299–321. Amer. Math. Soc., Providence, RI, 1989.

[22] R. Askey. A look at the Bateman project. In *The mathematical legacy of Wilhelm Magnus: groups, geometry and special functions (Brooklyn, NY, 1992)*, volume 169 of *Contemp. Math.*, pages 29–43. Amer. Math. Soc., Providence, RI, 1994.

[23] R. Askey. Vietoris's inequalities and hypergeometric series. In *Recent progress in inequalities (Niš, 1996)*, volume 430 of *Math. Appl.*, pages 63–76. Kluwer Acad. Publ., Dordrecht, 1998.

[24] R. Askey. Some elementary uses of complex numbers. *Computational Methods and Function Theory*, 10(2):579–584, 2010.

[25] R. Askey and N. H. Bingham. Gaussian processes on compact symmetric spaces. *Z. Wahrscheinlichkeitstheorie und Verw. Gebiete*, 37(2):127–143, 1976/77.

[26] R. Askey, R. DeVore, G. Freud, J. Musielak, J. Peetre, and T. Popoviciu. Proposed problems. In *Proceedings of the Conference on the Constructive Theory of Functions (Approximation Theory) (Budapest, 1969)*, pages 533–538, 1972.

[27] R. Askey, J. Fitch, and G. Gasper. On a positive trigonometric sum. *Proceedings of the American Mathematical Society*, 19:1507, 1968.

[28] R. Askey and G. Gasper. Positive Jacobi polynomial sums. II. *American Journal of Mathematics*, 98(3):709–737, 1976.

[29] R. Askey and I. I. Hirschman, Jr. Mean summability for ultraspherical polynomials. *Mathematica Scandinavica*, 12:167–177, 1963.

[30] R. Askey and M. Ismail. Recurrence relations, continued fractions, and orthogonal polynomials. *Memoirs of the American Mathematical Society*, 49(300):iv+108, 1984.

[31] R. Askey and M. E. H. Ismail. Permutation problems and special functions. *Canadian Journal of Mathematics. Journal Canadien de Mathématiques*, 28(4):853–874, 1976.

[32] R. Askey and M. E. H. Ismail. The very well poised ${}_6\psi_6$. *Proceedings of the American Mathematical Society*, 77(2):218–222, 1979.

[33] R. Askey and M. E. H. Ismail. A generalization of ultraspherical polynomials. In *Studies in pure mathematics*, pages 55–78. Birkhäuser, Basel, 1983.

[34] R. Askey and G. Patruno. Problems and Solutions: Solutions of Advanced Problems: 6514. *American Mathematical Monthly*, 94(10):1012–1014, 1987.

[35] R. Askey and R. Roy. More q–beta integrals. *The Rocky Mountain Journal of Mathematics*, 16(2):365–372, 1986.

[36] R. Askey, R. E. Shafer, E. Kronheimer, and A. Lupas. Problems and Solutions: Advanced Problems: 6514–6517. *American Mathematical Monthly*, 93(4):304–305, 1986.

[37] R. Askey and J. Steinig. Some positive trigonometric sums. *Transactions of the American Mathematical Society*, 187:295–307, 1974.

[38] R. Askey and S. Wainger. Mean convergence of expansions in Laguerre and Hermite series. *American Journal of Mathematics*, 87:695–708, 1965.

[39] R. Askey and J. Wilson. Some basic hypergeometric orthogonal polynomials that generalize Jacobi polynomials. *Memoirs of the American Mathematical Society*, 54(319):iv+55, 1985.

[40] A. Baernstein II, D. Drasin, P. Duren, and A. Marden, editors. *The Bieberbach conjecture*, volume 21 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI, 1986.

[41] H. Bavinck. On absolute convergence of Jacobi series. *Journal of Approximation Theory*, 4:387–400, 1971.

[42] H. Bavinck. *Jacobi series and approximation*. Mathematisch Centrum, Amsterdam, 1972. Mathematical Centre Tracts, No. 39.

[43] B. C. Berndt. The quarterly reports of S. Ramanujan. *American Mathematical Monthly*, 90(8):505–516, 1983.

[44] B. C. Berndt. *Ramanujan's notebooks. Part I*. Springer–Verlag, New York, 1985. With a foreword by S. Chandrasekhar.

[45] B. C. Berndt. *Ramanujan's notebooks. Part II*. Springer–Verlag, New York, 1989.

[46] B. C. Berndt. *Ramanujan's notebooks. Part III*. Springer–Verlag, New York, 1991.

[47] B. C. Berndt. *Ramanujan's notebooks. Part IV*. Springer–Verlag, New York, 1994.

[48] B. C. Berndt. *Ramanujan's notebooks. Part V*. Springer–Verlag, New York, 1998.

[49] J. Bertrand. *Traité de Calcul Différentiel et de Calcul Intégral*. Gauthier–Villars, Paris, 1870. Volume 2.

[50] D. Bessis and P. Moussa. Orthogonality properties of iterated polynomial mappings. *Communications in Mathematical Physics*, 88(4):503–529, 1983.

[51] F. Beukers and G. Heckman. Monodromy for the hypergeometric function ${}_nF_{n-1}$. *Inventiones Mathematicae*, 95(2):325–354, 1989.

[52] G. Bhatnagar and M. J. Schlosser. A partial theta function Borwein conjecture. *Annals of Combinatorics*, 23:561–572, 2019. *arXiv:1902.04447*.

[53] N. H. Bingham. Tauberian theorems for Jacobi series. *Proceedings of the London Mathematical Society. Third Series*, 36(2):285–309, 1978.

[54] J. W. Bober. Factorial ratios, hypergeometric series, and a family of step functions. *Journal of the London Mathematical Society. Second Series*, 79(2):422–444, 2009.

[55] D. C. Brown and S. W. Davies. Financing Efficiency of Securities-Based Crowdfunding. *SSRN Electronic Journal*, pages 1–64, 2019. (and pages 1–42 in the Internet Appendix).

[56] I. Cherednik. Double affine Hecke algebras and Macdonald's conjectures. *Annals of Mathematics. Second Series*, 141(1):191–216, 1995.

[57] L. de Branges. A proof of the Bieberbach conjecture. *Acta Mathematica*, 154(1–2):137–152, 1985.

[58] N. G. de Bruijn. *Asymptotic methods in analysis*. Dover Publications, Inc., New York, third edition, 1981.

[59] *NIST Digital Library of Mathematical Functions*. *http://dlmf.nist.gov*, Release 1.0.27 of 2020-06-15. F. W. J. Olver, A. B. Olde Daalhuis, D. W. Lozier, B. I. Schneider, R. F. Boisvert, C. W. Clark, B. R. Miller, B. V. Saunders, H. S. Cohl, and M. A. McClain, eds.

[60] P. Duren, R. A. Askey, H. M. Edwards, and U. C. Merzbach, editors. *A Century of Mathematics in America. Part III*, volume 3 of *History of Mathematics*. American Mathematical Society, Providence, RI, 1989.

[61] P. Duren, R. A. Askey, and U. C. Merzbach, editors. *A Century of Mathematics in America. Part I*, volume 1 of *History of Mathematics*. American Mathematical Society, Providence, RI, 1988.

[62] P. Duren, R. A. Askey, and U. C. Merzbach, editors. *A Century of Mathematics in America. Part II*, volume 2 of *History of Mathematics*. American Mathematical Society, Providence, RI, 1989.

[63] J. Edwards. *A Treatise on the Integral Calculus, with applications, examples and problems*, volume 1. Chelsea Publishing Company, New York, NY, 1921.

[64] J. Edwards. *A Treatise on the Integral Calculus, with applications, examples and problems*, volume 2. Chelsea Publishing Company, New York, NY, 1922.

[65] Leon Ehrenpreis and Robert C. Gunning, editors. *Theta functions—Bowdoin 1987. Part 1*, volume 49 of *Proceedings of Symposia in Pure Mathematics*. American Mathematical Society, Providence, RI, 1989.

[66] Leon Ehrenpreis and Robert C. Gunning, editors. *Theta functions—Bowdoin 1987. Part 2*, volume 49 of *Proceedings of Symposia in Pure Mathematics*. American Mathematical Society, Providence, RI, 1989.

[67] A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi. *Tables of Integral Transforms. Vols. 1-2*. McGraw-Hill Book Company, Inc., New York-Toronto-London, 1954.

[68] A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi. *Higher Transcendental Functions. Vols. 1-3*. Robert E. Krieger Publishing Co. Inc., Melbourne, Fla., 1981.

[69] L. Euler, June 1750. *Letter from L. Euler to C. Goldbach on June 9, 1750*.

[70] L. Euler. *Institutiones Calculi Integralis*. Impensis Academiae Imperialis Scientiarum, St. Petersburg, 1768.

[71] L. Euler. De mirabilibus proprietatibus unciarum, quae in evolutione binomii ad potestatem quamcunqua evecti occurrunt. *Acta Academiae Scientarum Imperialis Petropolitinae*, 5:74–111, 1784. presented to the St. Petersburg Academy on May 13, 1776.

[72] G. Gasper. Projection formulas for orthogonal polynomials of a discrete variable. *Journal of Mathematical Analysis and Applications*, 45:176–198, 1974.

[73] G. Gasper. Positivity and special functions. In *Theory and application of special functions (Proc. Advanced Sem., Math. Res. Center, Univ. Wisconsin, Madison, Wis., 1975)*, pages 375–433. Math. Res. Center, Univ. Wisconsin, Publ. No. 35, 1975.

[74] G. Gasper. Positive sums of the classical orthogonal polynomials. *SIAM Journal on Mathematical Analysis*, 8(3):423–447, 1977.

[75] G. Gasper, M. E. H. Ismail, T. Koornwinder, P. Nevai, and D. Stanton. The mathematical contributions of Richard Askey. In *q-series from a contemporary perspective (South Hadley, MA, 1998)*, volume 254 of *Contemp. Math.*, pages 1–18. Amer. Math. Soc., Providence, RI, 2000.

[76] G. Gasper and M. Rahman. *Basic hypergeometric series*, volume 35 of *Encyclopedia of Mathematics and its Applications*. Cambridge University Press, Cambridge, 1990. With a foreword by Richard Askey.

[77] G. Gasper and M. Rahman. *Basic hypergeometric series*, volume 96 of *Encyclopedia of Mathematics and its Applications*. Cambridge University Press, Cambridge, second edition, 2004. With a foreword by Richard Askey.

[78] G. Gasper and M. Rahman. Some systems of multivariable orthogonal Askey–Wilson polynomials. In *Theory and applications of special functions*, volume 13 of *Dev. Math.*, pages 209–219. Springer, New York, 2005.

[79] L. Habsieger. La q–conjecture de Macdonald–Morris pour G_2. *Comptes Rendus des Séances de l'Académie des Sciences. Série I. Mathématique*, 303(6):211–213, 1986.

[80] E. Heine. *Handbuch der Kugelfunctionen, Theorie und Anwendungen (volume 2)*. Druck und Verlag von G. Reimer, Berlin, 1881.

[81] M. D. Hirschhorn. The AM–GM inequality. *The Mathematical Intelligencer*, 29(4):7–7, Sep 2007.

[82] M. D. Hirschhorn. A surprising appearance of $\zeta(3)$. *The Mathematical Gazette*, 95(533):313–313, 2011.

[83] M. D. Hirschhorn. A constant in a formula of Ramanujan. *The Mathematical Gazette*, 97(539):276–277, 2013.

[84] T. H. Koornwinder. *Dick Askey's positive addition to Amsterdam*, slides of a lecture at the conference *Dick Askey's 80th*, Madison, Wisconsin, 6–7 December 2013. *https://staff.fnwi.uva.nl/t.h.koornwinder/art/sheets/askey80.pdf*, 2013.

[85] T. H. Koornwinder. Dual addition formulas associated with dual product formulas. In *Frontiers in Orthogonal Polynomials and q-Series*, chapter 19, pages 373–392. World Scientific Publishing, Hackensack, NJ, 2018. Zuhair Nashed and Xin Li, editors, *arXiv:1607.06053v4*.

[86] S. I. Kryuchkov, S. K. Suslov, and J. M. Vega-Guzmán. The minimum-uncertainty squeezed states for atoms and photons in a cavity. *Journal of Physics B: Atomic, Molecular and Optical Physics*, 46(10), 5 2013. (IOP=Institute Of Physics SELECT and HIGHLIGHT for 2013).

[87] S. F. Lacroix. *Traité du Calcul Différentiel et de Calcul Intégral*, volume 2. V. Courcier, Paris, second edition, 1814.

[88] E. Landau. *Collected works. Vol. 1*. Thales-Verlag, Essen, 1985. With a contribution in English by G. H. Hardy and H. Heilbronn, Edited and with a preface in English by L. Mirsky, I. J. Schoenberg, W. Schwarz and H. Wefelscheid.

[89] Z.-G. Liu. A q-series expansion formula and the Askey-Wilson polynomials. *Ramanujan Journal. An International Journal Devoted to the Areas of Mathematics Influenced by Ramanujan*, 30(2):193–210, 2013.

[90] Z.-G. Liu. A q-summation formula, the continuous q-Hahn polynomials and the big q-Jacobi polynomials. *Journal of Mathematical Analysis and Applications*, 419(2):1045–1064, 2014.

[91] Z.-G. Liu. Askey-Wilson polynomials and a double q-series transformation formula with twelve parameters. *Proceedings of the American Mathematical Society*, 147(6):2349–2363, 2019.

[92] D. W. Lozier. Toward a Revised NBS Handbook of Mathematical Functions. *NIST Technical Reports*, NISTIR 6072:1–8, September 1997.

[93] I. G. Macdonald. Some conjectures for root systems. *SIAM Journal on Mathematical Analysis*, 13(6):988–1007, 1982.

[94] I. G. Macdonald. *Symmetric functions and Hall polynomials*. Oxford Classic Texts in the Physical Sciences. The Clarendon Press, Oxford University Press, New York, second edition, 2015. With contribution by A. V. Zelevinsky and a foreword by Richard Stanley, Reprint of the 2008 paperback edition [MR1354144].

[95] R. S. Maier. Extensions of the classical transformations of the hypergeometric function ${}_3F_2$. *Advances in Applied Mathematics*, 105:25–47, 2019.

[96] J. Meixner and F. W. Schäfke. *Mathieusche Funktionen und Sphäroidfunktionen mit Anwendungen auf physikalische und technische Probleme*. Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen mit besonderer Berücksichtigung der Anwendungsgebiete, Band LXXI. Springer-Verlag, Berlin, 1954.

[97] W. Miller, Jr. *Symmetry and separation of variables*. Addison-Wesley Publishing Co., Reading, Mass.-London-Amsterdam, 1977. With a foreword by Richard Askey, Encyclopedia of Mathematics and its Applications, Vol. 4.

[98] W. G. Morris. *Constant term identities for finite and affine root systems: conjectures and theorems*. PhD thesis, University of Wisconsin-Madison, Madison, Wisconsin, 1982. iii+142 pages.

[99] P. G. Nevai and J. S. Dehesa. On asymptotic average properties of zeros of orthogonal polynomials. *SIAM Journal on Mathematical Analysis*, 10(6):1184–1192, 1979.

[100] A. Okounkov. Binomial formula for Macdonald polynomials and applications. *Mathematical Research Letters*, 4(4):533–553, 1997.

[101] A. Okounkov. BC-type interpolation Macdonald polynomials and binomial formula for Koornwinder polynomials. *Transformation Groups*, 3(2):181–207, 1998.

[102] A. Okounkov and G. Olshanski. Shifted Jack polynomials, binomial formula, and applications. *Mathematical Research Letters*, 4(1):69–78, 1997.

[103] F. Pollaczek. *Sur une généralisation des polynomes de Jacobi*. Mémorial des Sciences Mathématiques, Fascicule 131. Gauthier-Villars, Paris, 1956.

[104] E. M. Rains. BC$_n$-symmetric polynomials. *Transformation Groups*, 10(1):63–132, 2005.

[105] E. M. Rains. BC_n-symmetric Abelian functions. *Duke Mathematical Journal*, 135(1):99–180, 2006.

[106] E. D. Rainville. *Special Functions*. The Macmillan Co., New York, 1960.

[107] S. Ramanujan. *Notebooks. Vols. 1, 2*. Tata Institute of Fundamental Research, Bombay, 1957.

[108] S. Ramanujan. *The lost notebook and other unpublished papers*. Springer-Verlag, Berlin; Narosa Publishing House, New Delhi, 1988. With an introduction by George E. Andrews.

[109] F. Rodriguez-Villegas. Integral ratios of factorials and algebraic hypergeometric functions. *Olberwolfach Reports*, 2:1814–1816, 2005.

[110] S. Roman. More on the umbral calculus, with emphasis on the q-umbral calculus. *Journal of Mathematical Analysis and Applications*, 107(1):222–254, 1985.

[111] H. Rosengren and S. O. Warnaar. Elliptic hypergeometric functions associated with root systems. *arXiv:1704.08406v3*, 2019.

[112] G.-C. Rota. *Indiscrete thoughts*. Modern Birkhäuser Classics. Birkhäuser Boston, Inc., Boston, MA, 2008. Reprint of the 1997 edition [Birkhäuser Boston, Boston, MA; MR1419503], With forewords by Reuben Hersh and Robert Sokolowski, Edited, with notes and an epilogue by Fabrizio Palombi.

[113] H. A. Rothe. *Systematisches Lehrbuch der Arithmetik*. Johann Ambrosius Barth, Leipzig, 1811.

[114] M. J. Schlosser. A Taylor expansion theorem for an elliptic extension of the Askey-Wilson operator. In *Special functions and orthogonal polynomials*, volume 471 of *Contemp. Math.*, pages 175–186. Amer. Math. Soc., Providence, RI, 2008.

[115] F. Schweins. *Analysis*. Mohr und Winter, Heidelberg, 1820.

[116] A. Selberg and S. Chowla. On Epstein's zeta-function. *Journal für die Reine und Angewandte Mathematik. [Crelle's Journal]*, 227:86–110, 1967.

[117] K. Soundararajan. Integral factorial ratios. *arXiv:1901.05133*, pages 1–31, 2019.

[118] K. Soundararajan. Integral factorial ratios: irreducible examples with height larger than 1. *arXiv:1906.06413*, pages 1–16, 2019.

[119] V. P. Spiridonov. On the elliptic beta function. *Rossiiskaya Akademiya Nauk. Moskovskoe Matematicheskoe Obshchestvo. Uspekhi Matematicheskikh Nauk*, 56(1(337)):181–182, 2001.

[120] V. P. Spiridonov. Rarefied elliptic hypergeometric functions. *Advances in Mathematics*, 331:830–873, 2018.

[121] V. P. Spiridonov. Superconformal indices, Seiberg dualities and special functions. *Physics of Particles and Nuclei*, 51:508–513, 2020.

[122] G. Szegő. *Collected papers. Vol. 1*. Contemporary Mathematicians. Birkhäuser, Boston, Mass., 1982. 1915–1927, Edited by R. A. Askey, Including commentaries and reviews by G. Pólya, P. C. Rosenbloom, R. A. Askey, L. E. Payne, T. Kailath and B. M. McCoy.

[123] G. Szegő. *Collected papers. Vol. 2*. Contemporary Mathematicians. Birkhäuser, Boston, Mass., 1982. 1927–1943, Edited by R. A. Askey.

[124] G. Szegő. *Collected papers. Vol. 3*. Contemporary Mathematicians. Birkhäuser, Boston, Mass., 1982. 1945–1972, Edited by R. A. Askey.

[125] G. Szegő. *Orthogonal polynomials*. American Mathematical Society Colloquium Publications, Vol. 23. Revised ed. American Mathematical Society, Providence, R.I., 1959.

[126] P. L. Tchebichef. Mémoire sur les nombres premiers. *Journal des Mathématiques Pures et Appliquées*, 17:366–390, 1852.

[127] K. Venkatachaliengar. *Development of elliptic functions according to Ramanujan*, volume 6 of *Monographs in Number Theory*. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2012. Edited, revised, and with a preface by Shaun Cooper.

[128] L. Vietoris. Über das Vorzeichen gewisser trigonometrischer Summen. *Österreichische Akademie der Wissenschaften, Mathematisch-Naturwissenschaftliche Klasse. Sitzungsberichte. Abteilung II*, 167:125–135, 1958.

[129] H. Volkmer. On some conjectures of Askey. *Asymptotic Analysis*, 27(3–4):321–334, 2001.

[130] C. Wang. An analytic proof of the Borwein Conjecture. *arXiv:1901.10886*, 2019.

[131] S. O. Warnaar and W. Zudilin. A q–rious positivity. *Aequationes Mathematicae*, 81(1–2):177–183, 2011.

[132] H. Widom and H. Wilf. Small eigenvalues of large Hankel matrices. *Proceedings of the American Mathematical Society*, 17:338–344, 1966.

[133] D. Zeilberger. A proof of the G_2 case of Macdonald's root system–Dyson conjecture. *SIAM Journal on Mathematical Analysis*, 18(3):880–883, 1987.

[134] D. Zeilberger. Proof of the refined alternating sign matrix conjecture. *New York Journal of Mathematics*, 2:59–68, 1996.